# Bubbles and Behavioral Finance

*Edward Chi-Ho Tang, Kai-Yin Woo,*
*Tai-Yuen Hon, Wing-Kwong Au,*
*Wing-Keung Wong and Hok-Fu Wu*


B P International

# Bubbles and Behavioral Finance

# Bubbles and Behavioral Finance

India ▪ United Kingdom

B P International

## Author(s)

**Edward Chi-Ho Tang**
Hong Kong Shue Yan University, Hong Kong.

**Kai-Yin Woo**
Department of Economics and Finance, Hong Kong Shue Yan University, Hong Kong.

**Tai-Yuen Hon**
Business, Economic and Public Policy Research Centre, Hong Kong Shue Yan University, Hong Kong.

**Wing-Kwong Au**
Department of Social Work, Hong Kong Shue Yan University, Hong Kong.

**Wing-Keung Wong**
Department of Finance, Fintech & Blockchain Research Center, and Big Data Research Center, Asia University, Taiwan, Department of Medical Research, China Medical University Hospital Business, Taiwan and Economic and Public Policy Research Centre, Hong Kong Shue Yan University, Hong Kong.

**Hok-Fu Wu**
Crowe (HK) Global Corporate Advisory Limited, Hong Kong.

**FIRST EDITION 2024**

**ISBN 978-81-973195-8-7 (Print)**
**ISBN 978-81-973195-0-1 (eBook)**
**DOI: 10.9734/bpi/mono/978-81-973195-8-7**

9 788197 319587

9 788197 319501

**Peer-Review Policy:** Advanced Open Peer Review policy has been followed for review. All chapters were thoroughly checked to prevent plagiarism. As per editorial policy, a minimum of two peer-reviewers reviewed the manuscript. After review and revision of the manuscript, the Book Editor approved the manuscript for final publication.

**Chapter 1**

*Approved by*

(1) Dr. Jongwha Chang, Texas Woman's University, College of Business, USA.

*Reviewers*

(1) Tugkan Arici, Gebze Technical University, Turkiye.
(2) Andrey Artemenkov, Ariel University, Israel.
(3) Aijaz Ahmad Bhat, India.
(4) Joseph Oyewale Oyedeji, Bells University of Technology, Nigeria.
(5) Rashesh Vaidya, Nepal Open University, Nepal.

**Chapter 2**

*Approved by*

(1) Dr. Luca Grilli, University of Foggia, Italy.

*Reviewers*

(1) Sanjay Kumar Jha, Lalit Narayan Mithila University, India.
(2) M. Kasi Mayan, India.
(3) Wei-Hsiung Chang, Nanhua University, Taiwan.

**Chapter 3**

*Approved by*

(1) Dr. Luca Grilli, University of Foggia, Italy.

*Reviewers*

(1) Ali Mohamed Awin, University of Tripoli. Libya.
(2) Bayu Adi Nugroho, Indonesia.

**Chapter 4**

*Approved by*

(1) Prof. Ayoub Azzayani, Mohammed V University, Rabat, Morocco.

*Reviewers*

(1) Sanjay Kumar Jha, Lalit Narayan Mithila University, India.
(2) Prakash Pinto, St Joseph Engineering College, India.
(3) Rashesh Vaidya, Nepal Open University, Nepal.

**Chapter 5**

*Approved by*

(1) Dr. Arzu Alvan Bozdereli, Cyprus Science University, Cyprus.

*Reviewers*

(1) Nemer Badwan, Palestine Technical University, Palestine.
(2) Bishwajeet Bhattacharjee, India.

# *Contents*

## PREFACE

This book has complied with two Journal papers and three working papers. We do not have any copyright issue in our book. This book includes Chapter 1 revisits the housing bubble by using right tailed ADF test and provides useful policy recommendations at the end. Chapter 2 reviews bubbles solutions to the Cagan hyperinflation models under rational expectations. Chapter 3 undertakes Marko-switching cointegration test of price and exchange rate bubbles. Chapter 4 attempts to identify and analyse the key factors that capture small investors' behaviour in the Hong Kong stock market. Chapter 5 investigates the macroeconomic fundamentals in the Hong Kong stock market. The authors are eager to get this book published and intend to maintain strong friendships. We are closely cooperating with each other and are more united than ever.

***Chapter 1***
Print ISBN: 978-81-973195-8-7, eBook ISBN: 978-81-973195-0-1

# Does Bubble Still Exist after COVID-19? Evidence from Hong Kong Housing Market

**Edward Chi-Ho Tang [a*]**

DOI: 10.9734/bpi/mono/978-81-973195-8-7/CH1

**Peer-Review History:**
This chapter was reviewed by following the Advanced Open Peer Review policy. This chapter was thoroughly checked to prevent plagiarism. As per editorial policy, a minimum of two peer-reviewers reviewed the manuscript. After review and revision of the manuscript, the Book Editor approved the manuscript for final publication. Peer review comments, comments of the editor(s), etc. are available here: https://peerreviewarchive.com/review-history/12411F-CH1

## ABSTRACT

Hong Kong housing is commonly known as one of the most unaffordable housing in the world. The housing bubble is formed due to low interest rates, a limited amount of housing supply and speculation. Even though the Hong Kong government launched countercyclical housing policies to stabilize the market, the inflating housing bubble was unstoppable. The situation continued until the emergence of COVID-19, which brought a huge negative shock to the market. This paper revisits the housing bubble by using the right-tailed ADF test and provides useful policy recommendations at the end.

*Keywords: COVID-19; housing bubble; countercyclical housing policies; housing market.*

## 1.1 INTRODUCTION

Traditional finance theories assume that the market is frictionless, and investors are making rational investment decisions. However, the real financial market is far from perfect. The reasons include information asymmetry among the market participants, transaction costs and psychological biases of investors. These imperfections would lead to mispricing of assets, which are not limited to traditional financial products such as stocks and bonds, but also cryptocurrencies like Bitcoin. Obviously, asset mispricing would bring significant consequences to investors, companies, and the broad economy. A typical example comes from Hong Kong during the Asian Financial Crisis. From January 1990 to March 1997, Hong Kong experienced a growth of 3.8% in real wages, 153.1% in the real Hang Seng Index, and 101.4% in real housing prices. This widened the income inequality and inflated asset bubbles, which further weakened the competitive

[a] *Hong Kong Shue Yan University, Hong Kong.*
**Corresponding author: E-mail: chtang@hksyu.edu;*

power of Hong Kong. More importantly, it attracted speculators who wished to make enormous profits from the exchange rate market and the stock market by adopting “double market play”. This strategy worked either if the pegged exchange rate system was abandoned or the interest rate went up. Finally, the government raised the interest rate and used official reserves to purchase stocks from the financial market, which protected both the linked exchange rate system and the local economy from collapse. However, the rise of interest rates led to the burst of the asset bubble, which then translated into a surge in the delinquency ratio from 0.84% in 1998 to 1.31% in 2000.

Starting in 2010, the Hong Kong housing market caught international attention again because of its “well-known” unaffordability[1]. With the quantitative easing adopted by the United States, a lot of funds flowed into the Hong Kong housing market, which resulted in market exuberance. In addition to the low-interest rate environment (Fig. 1.1), it makes it easier for homebuyers to meet the debt servicing ratio and pass the stress test in mortgage financing, which inflated the second housing bubble in Hong Kong.

Another possible reason supporting the recent housing bubble is related to the small amount of housing supply. From Fig. 1.2, it is clearly shown that the new housing completions were kept at a low level during the 2010s[2]. With the expansion in population, it creates a shortage in housing. Potential buyers had to compete with others for limited supply. For the first-hand market, it was common to observe that all houses were sold out quickly on the first day of sale. At the same time, the second-hand market became the sellers’ market, where negotiation of a downward price became much more difficult.

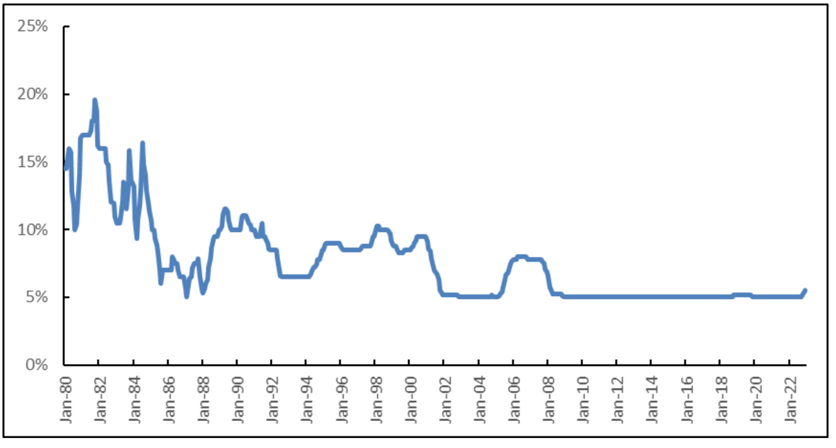



**Fig. 1.1. Best lending rate**
*Source: Census and Statistics Department*

*[1] According to Demograhpia (2023), Hong Kong housing is the least affordable for 13th straight years.*
*[2] See Leung et al. (2020a, 2020b) and Leung and Tang (2023) for more discussion.*

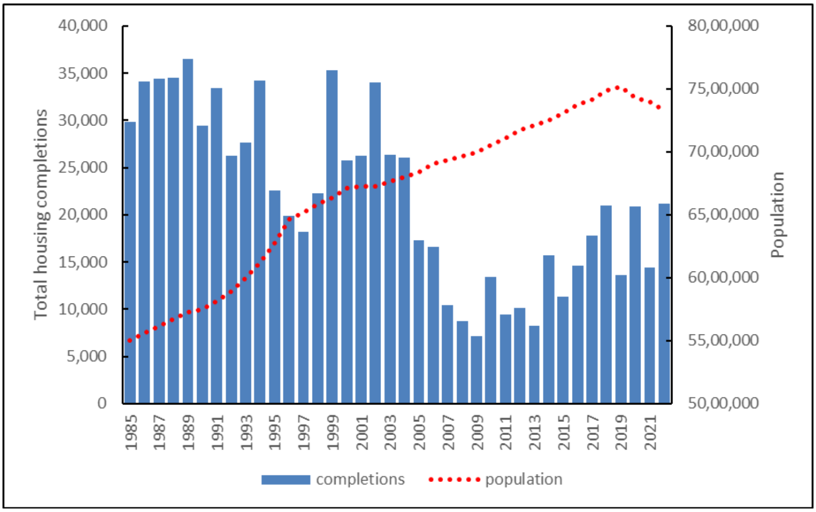



**Fig. 1.2. Total housing completions and population**
_Source: Rating and Valuation Department_

Even though the government introduced countercyclical housing policies in 2012, housing prices kept surging, which is supported by Gyourko and Molloy (2015). At the same time, speculative behavior in the housing market remained strong. Yiu et al. (2013) find that the bubble mainly comes from the mass market. Hence, society created a belief that housing is a kind of good investment as its price will keep surging. It further motivated new homebuyers to make use of high leverage in home purchases[3], and thus housing bubble is accompanied by high trading volume (Barberis et al., 2018). This would generate a self-fulfilling cycle, which poses further risks to the whole financial system (Anundsen et al, 2016).

Furthermore, prior studies concerned the effectiveness of macroprudential policies in the bubble-creation economy. Luangaram and Thepmongkol (2022) find that “restrictive policies tend to be more effective in dampening asset-price bubbles in economies that have a high degree of financial depth.” Wong et al. (2021) show that credit-tightening policies in Hong Kong were able to curb the house price growth in the high-price segment, while transaction taxes could not. It is because homebuyers find it easier to avoid the extraordinary stamp duties. Similarly, Deng et al. (2024) found that mortgage-tightening measures in Hong Kong effectively cooled down the overheated market, while tax-driven policies suppressed trading activity and triggered price volatilities across submarkets. By revisiting the Hong Kong housing bubble, our paper would complement this strand of literature and discuss whether the countercyclical housing policies should be continued.

_[3] Under the current Mortgage Insurance Programme, for eligible properties with the property value up to HK$10 million (which is usually regarded as “starter home”), the maximum loan-to-value ratio is 90%._

With the outbreak of COVID-19, the financial market entered into a new phase. It has become popular among researchers to study the effect of the pandemic on the macroeconomy. Anundsen et al. (2023) showed that COVID-19 leads to lower house seller confidence and more exploitative bidding. In the case of Hong Kong, the prolonged quarantine rules and strict preventive measures reduce Hong Kong investor's sentiment. Therefore, it is interesting to determine if the market exuberance persists to drive the housing bubble. Our paper will use the right-tailed ADF test to revisit the housing bubble in Hong Kong. It will be useful in the following sense:

- Discussing the risks associated with the housing bubble burst: The housing market is dominated by amateur investors (Glaeser and Nathanson, 2015), and they are susceptible to market adjustments. Especially, even though the housing is in a downturn, they tend not to sell the property to minimize the loss, which further threatens their financial conditions. Our empirical result will provide a reference on the starting and ending month of a housing bubble and call for the investors' attention to the risks associated with purchasing properties at the hike.
- Investigating the developers' strategies in constructing and selling properties: During the housing boom, developers are willing to bid the land at a high price and sell the completed properties at a slow pace, which transmits a favorable market signal that the housing market will surge. However, when the housing market is in a downturn, developers who do not have a deep pocket will rush to sell the completed properties, which speeds up the decline in housing prices. By the end of the paper, it uses a recent example to talk about the considerations of buying properties through highly leveraged mortgages.
- Formulating housing policies: In the past decade, the government introduced a number of countercyclical housing policies to cool down the overheated housing market, including double stamp duty, special stamp duty, and buyer stamp duty[4]. Under the reversal of the housing market, there are some debates on the timing of removing these extraordinary measures. Our results will provide a basis for policy discussions.

In a nutshell, the main objective of the paper is to complement the existing literature by checking if the recent housing bubble burst during the COVID-19 pandemic. If so, it further explains the risks and consequences associated with the burst of a bubble, such as:

- Will the burst of the housing bubble result in a surge in negative equity cases?
- How does the burst of the housing bubble change the developers' strategies in house selling and land bidding?
- What are the considerations of using a "Stage Payment Plan" to purchase a flat in the primary market?

---

*[4] See Leung (2015) and Tang (2021) on how taxes affect real estate markets in Hong Kong.*

The rest of the chapter will be organized as follows. Section 2 presents the literature review and section 3 describes the data and methodology. Section 4 provides the empirical results and discussion, while section 5 shows the results of the robustness check. The final section will discuss the risk considerations of housing investment.

## 1.2 LITERATURE REVIEW

Asset bubbles, which are treated as a kind of mispricing in the market, have been extensively studied in previous literature. Fama (1965) mentioned that "bubbles are typically defined as periods in which asset prices run well above or below the intrinsic value", and Case and Shiller (2003) suggested that "excessive public expectations of future price increase causes prices to be temporarily elevated". When the asset bubble bursts, it leads to a sharp decline in asset prices and brings significant losses to investors. Credit tightens as lenders and banks are unwilling to lend out the money. The whole economy turns into recession quickly as a result.

Bubbles can exist in various investment assets and commodities, and the reasons for bubble formation are different. Take the Dutch tulip mania that happened in the 1630s as an indicative example to illustrate asset bubbles (Garber, 2000). By that time, it attracted a lot of speculation on tulips because they were new to Europe. People were purchasing the tulip bulbs at a higher price and reselling them at an even higher price to make a profit. Clearly, the prices were exacerbated to an unsustainable level. When the asset bubble burst, no one was willing to pay such a high price, causing a sharp fall in tulip prices. Another example is the Dot.com bubble in the late 1990s[5]. The market viewed the internet or technology industries as having a very high earnings potential, and therefore the bubble was fueled by a surge in investment in internet companies. However, these internet companies were immature and did not have a comprising business model. In some worst cases, companies pretended to engage in internet business by setting up a website only. At the end, a lot of companies failed to generate revenue or launch a product successfully. Investors were losing confidence in investing in technology companies, which caused a large drop in share prices, and further translated into economic recession.

More importantly, as the financial markets are getting more integrated, the negative consequences arising from a burst of asset bubble can easily be spread out to other markets. One typical example is related to the Global Financial Crisis in 2008[6]. The starting point of this crisis came from credit exuberance (Jorda et al, 2016), where lenders were very loose in processing mortgage applications. Homebuyers with poor credit histories were still successful in obtaining mortgages. The mortgages were then packaged together and sold to investors in the form of mortgage-backed securities (MBS), allowing the lenders to receive the cash for initiating the mortgage business again. As the lenders failed to

---

[5] *Interested readers can refer to McAleer et al. (2016) and Bai et al. (2015).*
[6] *See Lean et al. (2015), Leung and Tang (2011), Zhu et al. (2019) for more discussion.*

perform due diligence in the mortgage approval process, a lot of borrowers found difficulties repaying mortgages when the interest rate increased from 1% to 6.25% during 2004-2006. MBS investors, who were misrepresented by the bankers and believed that the investment was relatively “safe” at first, suffered tremendous losses. In 2009, the worldwide economy slumped quickly. U.S. GDP was reduced by 2.8% whereas Europe's counterpart was shrunk by 4.5%. Farmer (2015) finds that “the stock market crash of 2008 really did cause the Great Recession”.

After the emergence of the Global Financial Crisis, a large body of literature exists to discuss the detection of bubbles. Phillips et al. (2011) provide a recursive test procedure for testing explosive behavior, and Homm and Breitung (2012) propose several tests (supADF, supDFC, supK, supBT and supB) for rational bubbles. Also, some studies are devoted to comparing the housing bubble in different areas. For example, Hui et al. (2012) adopted a time-varying risk model to investigate the housing bubble in Guangzhou and Shenzhen, while Teng et al. (2013) used the state-space model to estate the sizes of a housing bubble in Taipei and Hong Kong. Lai and Van Order (2020) find that “the experience of boom-bubble-bust in the US market cannot be directly applied to China”. Our paper will be based on the work by Philip et al. (2015), which provides econometric detection mechanisms for identifying the dates of the bubble. This piece of work is widely cited in housing bubble-related research, such as Bangura and Lee (2020), Li et al. (2021), Andre et al. (2022), Tang (2017) and others.

Recently, a lot of attention has been paid to the effect of the COVID-19 pandemic on the economic and financial environment. For example, Ulku et al. (2023) argued that COVID-19 formed a negative bubble in the stock market and led to substantial wealth transfers among investor types, Ji et al. (2022) found that the global stock markets performed poorly during the pandemic, Gharib et al. (2021) found that a bilateral contagion effect of bubbles in oil and gold markets during COVID-19, Ding et al. (2022) studied how COVID-19 affects the corporate sector at different stages of COVID-19 outbreak, and Chong et al. (2020) proposed that “the slow growth, the sluggish recovery of trade and the cross-country transmission of the unemployment rate are three significant risk factors that ASEAN economies are faced with”. Besides, researchers are working on models to predict the confirmed cases (Tajmouati et al., 2022; Tuan et al., 2022), as well as macro indicators (Safi et al, 2022; Foroni et al, 2020). In relation to the investors’ behavior, Bourdeau-Brien and Kryzanowski (2020) found that “natural disasters cause a statistically and economically significant increase in risk aversion at the local level”, Brown et al. (2018) show that “being struck by an extreme event substantially changed individuals’ risk perceptions as well as their beliefs about the frequency and magnitude of future shocks”, and Sun et al. (2021) studied if investor sentiment, driven by coronavirus-related news and economic-related announcements, is priced in the medical portfolios.

Our paper will complement the existing literature that COVID-19 is a systematic risk that leads to the burst of the housing bubble in Hong Kong. It also calls for attention among market participants about the issue of the asset bubble. A thorough understanding of the asset bubbles allows people to make informed investment decisions and helps the government to take appropriate measures for cooling the overheated market, thus promoting a more stable economic and financial environment.

## 1.3 DATA AND METHODOLOGY

This paper aims to study the housing bubble in Hong Kong. It stems from the theoretical background that, controlling the information set at time t ($\Omega_t$), the housing price at time *t* ($P_t$) should be equal to the sum of the rent at time t ($R_t$) and the present value of its price in the next period. The mathematical formula is:

$$P_t = E(R_t|\Omega_t) + E\left(\frac{P_{t+1}}{D_t}|\Omega_t\right)$$

where $D_t$ is the discount factor.

Through replacing $P_{t+1}$ on the right-hand side, it gives

$$P_t = E(R_t|\Omega_t) + E\left(\frac{R_{t+1}}{D_t}|\Omega_t\right) + E\left(\frac{P_{t+2}}{D_t D_{t+1}}|\Omega_t\right)$$

Therefore, by iteration, the housing price at time *t* can be decomposed into two parts, including the “fundamental” value and the transversality condition:

$$P_t = \sum_{i=0}^{\infty} E\left(\frac{R_{t+i}}{D_{t+i}^i}|\Omega_t\right) + \lim E\left(\frac{P_{t+1+i}}{D_{t+1+i}^i}|\Omega_t\right)$$

When the transversality condition approaches to zero, it means that the observed housing price equals to the fundamental value, which is represented by the present value of the perpetual stream of rents received from the house. In contrast, if the housing bubbles exist, it suggests that homebuyers are paying a price higher than the fundamental value, and they expect to be compensated for overpayment by the expected appreciation of the bubble component. So, bubble testing is in fact a test of the transversality condition.

Our paper collects the real housing price (*RHP*), which has been deflated by the consumer price index (A), for the period from 1983Q4 (which is the starting quarter of the linked exchange rate system in Hong Kong) to 2023Q1. The data covers important events such as the Asian Financial Crisis[7], the outbreak of SARS, the Global Financial Crisis and the COVID-19 pandemic. Compared to the prior studies on bubble testing, the usage of real variables in our paper

[7] *Refer to Kung and Wong (2009) for more details.*

provides a more accurate picture of economic activity and allows meaningful comparisons over time[8]. The descriptive statistics are shown in Table 1.1.

**Table 1.1. Summary statistics for the real housing price**

| Mean | Standard deviation | Skewness | Kurtosis |
|---|---|---|---|
| 1.955 | 1.016 | 0.645 | 2.151 |

*Source: Author's calculations*

To begin, the paper conducts a Johansen co-integration test between real housing prices and real rent. This test mainly examines the $\pi$ matrix in the vector error correction model:

$$\Delta \mathrm{y_t} = \pi \mathrm{y_{t-k}} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \cdots + \Gamma_{\mathrm{k-1}} \Delta y_{t-(k-1)} + u_t$$

where $\pi = \left(\sum_{i=1}^{k} \beta_i\right) - I_g$ and $\Gamma_\mathrm{i} = \left(\sum_{j=1}^{i} \beta_j\right) - I_g$. If the rank (r) of the $\pi$ matrix is significantly different from zero, it implies that real housing price and real rent are cointegrated and hence precludes the formation of a housing bubble (Abraham & Hendershott, 1996; Bangrua & Lee, 2020; Meen, 2002). The test is conducted in the following sequence:

$$H_0: r = 0 \ versus \ H_1: 0 < r \leq 2$$
$$H_0: r = 1 \ versus \ H_1: r = 2$$

After confirming that no cointegration exists, it proceeds to detect whether the bubble has existed in the Hong Kong housing market. It adopts the bubble testing technique proposed by Phillips, Shi and Yu (2015). Referring to the following equation,

$$RHP_t = \mu + \delta RHP_{t-1} + \sum_{i=1}^{p} \phi_i \Delta RHP_{t-i} + \varepsilon_t$$

the core idea of this technique is to test the null hypothesis of a unit root ($\delta$ = 1) against the alternative hypothesis of a mildly explosive autoregressive coefficient ($\delta$ > 1). Formally, we test

$$H_0: \delta = 1$$
$$H_1: \delta > 1$$

There are four different methods of testing the bubble. The simplest method is to set the window size ($r_0$) to 1 and hence make use of the full normalized sample $[r_1, r_2]$ for estimating $\delta$. This will be the right-tailed version of the standard ADF unit root test. When the ADF statistic is greater than the critical value, it suggests that a housing bubble is present in the market.

[8] *See Leung et al. (2006) for more discussion.*

The second method is to use the rolling window approach. It specifies a fixed size of window in calculating the ADF statistic. When it moves one step forward, the window's starting point and ending point will be incremented by one observation, and hence it produces a new ADF statistic. The rolling ADF (RADF) statistic will be defined as the supremum ADF statistic among all possible windows.

The third one will be the recursive approach. The estimation procedure comes with a fixed starting point and an expanding window. When the regression is recursively estimated, the starting point stays the same while the ending point is incremented by one observation, and hence produces a series of ADF statistics. The SADF statistic is defined as the supremum of the ADF statistic among all windows:

$$\mathrm{SADF}(\mathrm{r}_0) = \sup_{\mathrm{r}_2 \in [\mathrm{r}_0, 1]} \{\mathrm{ADF}_{\mathrm{r}_2}\}$$

The last method is the generalized SADF (GSADF). The approach is flexible in that both the starting point and the estimated windows can be varied. The GSADF statistic is defined as:

$$\mathrm{GSADF}(\mathrm{r}_0) = \sup_{\substack{\mathrm{r}_2 \in [\mathrm{r}_0, 1] \\ \mathrm{r}_1 \in [0, \mathrm{r}_2 - \mathrm{r}_0]}} \{\mathrm{ADF}_{\mathrm{r}_1}^{\mathrm{r}_2}\}$$

One additional benefit of using the last two methods is that it allows us to determine the start and the end of the bubbles. The idea is to compare the sequence of ADF statistics with the critical value. When the ADF statistic first crosses the critical value from below, it represents the start of the bubble. Oppositely, when the ADF statistic first crosses the critical value from above, it suggests the end of the bubble.

## 1.4 RESULTS AND DISCUSSION

Before conducting bubble testing, it is important to determine whether a long-run relationship exists between real housing prices and real rent. Table 1.2 shows that the trace statistic and max-eigenvalue statistic are smaller than the corresponding 5% critical value, suggesting that the null hypothesis of no cointegration is accepted. In other words, the increase in real house prices is not accompanied by the real rent.

**Table 1.2. Results of Johansen cointegration test**

| Null hypothesis | Trace statistic | 5% critical value (Trace test) | Max-eigenvalue statistic | 5% critical value (Max-eigenvalue test) |
|---|---|---|---|---|
| r = 0 | 13.43 | 15.49 | 12.23 | 14.26 |
| r = 1 | 1.40 | 3.84 | 1.40 | 3.84 |

*Source: Author's calculations*

After that, the paper uses four different methods to check the existence of the housing bubble (Table 1.3). The first method uses the full sample in estimation. The ADF statistic is 10% significant, suggesting the occurrence of the housing bubble. For the other three methods, the paper follows Tang (2017) to use a window size of 0.3. All results reject the null hypothesis at a 5% level and confirm that $\delta$ is a mildly explosive autoregressive coefficient.

**Table 1.3. Empirical results of right-tailed ADF test**

| | **ADF statistic** | **$H_0$: Real housing price has a unit root** |
|---|---|---|
| Method 1: RTADF | 1.197 * | Rejected |
| Method 2: RADF (window size = 0.3) | 3.001 ** | Rejected |
| Method 3: SADF (window size = 0.3) | 3.050 ** | Rejected |
| Method 4: GSADF (window size = 0.3) | 3.223 ** | Rejected |

*Note: ** and * denote 5% and 10% statistical significance respectively.*
*Source: Author's calculations*

Next, our paper uses SADF and GSADF to detect the bubble period. Followed by Tang (2017), when the ADF statistic exceeds the critical value for more than 2 quarters (≈ log (158)), we would identify it as a housing bubble. Fig. 1.3a presents the results of SADF, where two housing bubbles (denoted by the shaded area) are found during the sampling period. The first housing bubble occurred during the first three quarters of 1997, where speculation in the pre-sale market and lenient borrowing standards were very common. Upon the arrival of the Asian Financial Crisis in 1998, the real housing price exhibited a significant drop of 38.5%. The situation continued to be worsened by the outbreak of SARS in 2003. The next housing bubble appeared in 2012Q2, which is mainly due to the quantitative ease in the United States. As Hong Kong's best lending rate is kept at a low level of 5%, it lessens the borrowing cost and attracts investors to the housing market, causing market exuberance. The housing investing success stories and the low-interest rate environment further attract followers to invest in the market, making herding behavior[9] another important reason for the formation of the housing bubble. Our empirical result shows that the bubble ended in 2022Q3, which is clearly the COVID-19 pandemic. In Fig. 1.3b, it presents the graphical results of GSADF. The result is slightly different from SADF, in which three bubbles occurred during the sampling period: 1995Q2 – 1997Q4; 2011Q1 – 2011Q3; 2012Q2 – 2022Q1 (Table 1.4). Nevertheless, both SADF and GSADF suggest that the recent housing bubble burst during the COVID-19 pandemic.

Our empirical results have important practical implications. First, during the bubble period, developers built more studio flats, which require a smaller lump sum of downpayment, to target singles or young families. However, the liquidity of the studio flat market is comparatively low, which makes it harder to resell the property in the market after the housing bubble bursts (Table 1.5). Second, the pandemic changes the landscape of the macroeconomy dramatically. As Hong Kong experienced SARS in 2003, people responded quickly when COVID-19 was confirmed to be spread around Hong Kong, such as wearing masks, using

*[9] See Lam et al. (2012), Munkh-Ulzii et al. (2018) and Wong (2020) for more discussion.*

sterilizers and maintaining social distancing. Unexpectedly, COVID-19 lasted for a much longer time and disrupted the financial markets. It causes widespread disruption to businesses and results in job losses. The reduction in income means people are less affordable in making home-purchasing decisions. Overall, this systematic risk had caused significant market volatility. Investors became pessimistic in their investment behavior. Third, the occurrence of the housing bubble led to homebuyers' heavy reliance on high-leveraged mortgage loans (with an LTV ratio of 60%-90%). Its ratio increased sharply from 7% in December 2018 to 29.5% in December 2020. Upon the burst of the housing bubble, it resulted in a subsequent rise in the negative equity cases, which is clearly shown in Fig. 1.4. After three-quarters of the bubble burst, the real housing price dropped by 12%. If families are using a "10% downpayment plan" to purchase a house at the hike, they will fall into the negative equity trap. While Christopher Hui, Secretary for Financial Services and the Treasury, replied to Legislative Council members that "banks, in general, will not request early repayments so long as the borrowers are able to make payment on their residential loans accordingly to schedule" (The Standard, 23 November 2023), it is worthy to note that negative equity is a necessary but not sufficient condition for mortgage default (Foote et al., 2008). If the borrowers cannot repay the full loan at the end, banks may have to write off a portion of the loan balance as a loss. Therefore, for the sake of financial stability, financial institutions are recommended to monitor the credit quality of the mortgage portfolio and exercise caution in extending new mortgage loans. Last, but not least, it brings to our attention that the policy lag may reduce the effectiveness of countercyclical housing policies in stabilizing the housing market. Particularly, after the burst of the housing bubble in 2022, the policies may over-correct the market and introduce unnecessary volatility in the market. Therefore, it is essential for the government to reconsider the continuation of countercyclical housing policies in a timely manner.

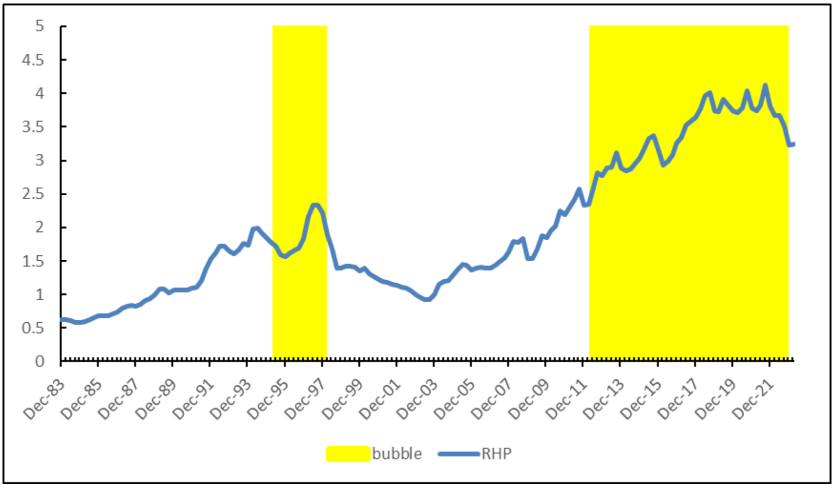



**Fig. 1.3a. Real housing price and housing bubble (SADF; window size = 0.3)**
*Sources: Rating and Valuation Department and author's calculations*

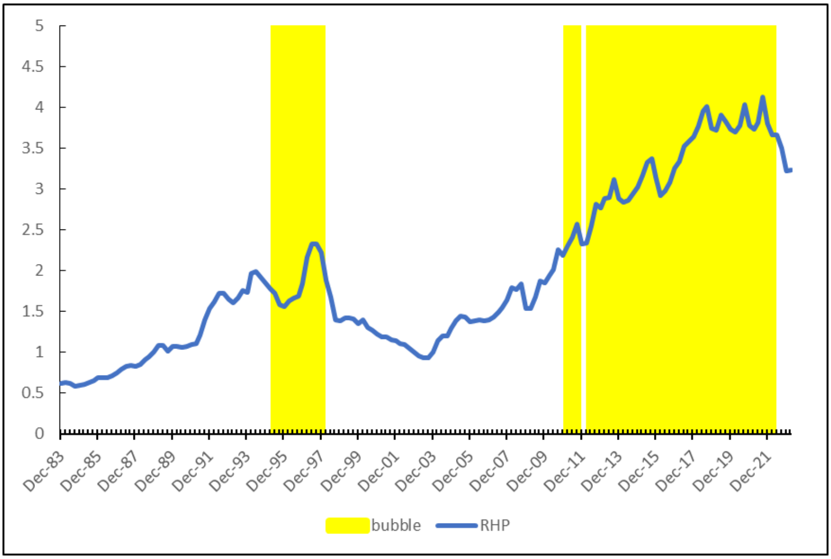



**Fig. 1.3b. Real housing price and housing bubble (GSADF; window size = 0.3)**
*Sources: Rating and Valuation Department and author's calculations*

**Table 1.4. Bubble periods**

| | **Bubble period** |
|---|---|
| Method 3: SADF (window size = 0.3) | 1995Q2 – 1997Q4; 2012Q2 – 2022Q3 |
| Method 4: GSADF (window size = 0.3) | 1995Q2 – 1997Q4; 2011Q1 – 2011Q3; 2012Q2 – 2022Q1 |

*Source: Author's calculations*

**Table 1.5. Studio flat transactions in Hong Kong**

| **Estate name** | **Completion date** | **Number of units** | **Number of transactions (first half of 2023)** |
|---|---|---|---|
| AVA 61 | 2020Q1 | 138 | 4 |
| AVA 262 | 2017Q2 | 88 | 0 |
| Novum East | 2019Q2 | 464 | 4 |
| One Prestige | 2018Q3 | 128 | 0 |
| Parker33 | 2017Q1 | 234 | 2 |
| Seven Victory Avenue | 2018Q4 | 250 | 4 |
| T Plus | 2019Q1 | 356 | 15 |

*Source: Centaline Agency*

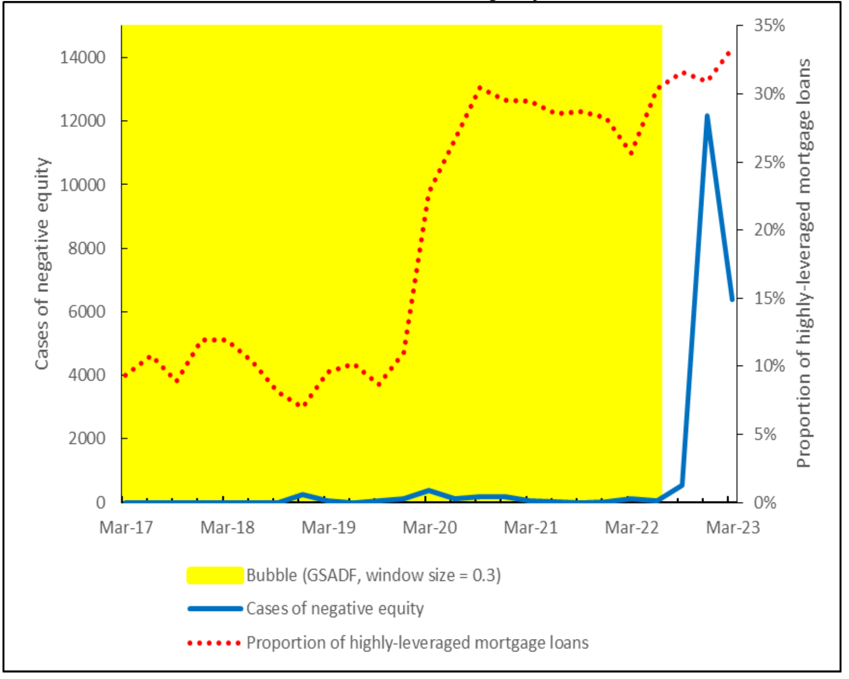



**Fig. 1.4. Cases of negative equity and proportion of highly-leveraged mortgage loans**

*Sources: Hong Kong Monetary Authority, Hong Kong Mortgage Corporation and author's calculations*

## 1.5 ROBUSTNESS CHECK

To check the robustness of our findings of the housing bubble test, we set the window size of 0.4 and re-run the right-tailed ADF tests. The empirical results are reported in Table 1.6. Similar to our baseline findings, the ADF statistics of the three tests are all statistically significant at a 5% level, suggesting the existence of housing bubbles. When we further examine the start and the end of housing bubbles, SADF suggests a bubble period of 2012Q2 – 2022Q3 and GSADF finds a similar bubble period of 2012Q2 – 2022Q2 (Table 1.7). These robust results offer further evidence to support our earlier argument that people's confidence in housing investment is still very strong at the beginning of the pandemic. The severity of COVID-19 brings border closures and quarantine rules that limit the freedom of movement and restrict regional and international cooperation. Such economic isolation results in investors' pessimism towards future economic conditions.

**Table 1.6. Robustness check of right-tailed ADF test**

| | **ADF statistic** | **$H_0$: Real housing price has a unit root** |
|---|---|---|
| Method 2: RADF (window size = 0.4) | 3.219 ** | Rejected |
| Method 3: SADF (window size = 0.4) | 2.980 ** | Rejected |
| Method 4: GSADF (window size = 0.4) | 3.219 ** | Rejected |

*Note: ** denotes 5% statistical significance.*
*Source: Author's calculations*

**Table 1.7. Bubble periods**

| | **Bubble period** |
|---|---|
| Method 3: SADF (window size = 0.4) | 2012Q2 – 2022Q3 |
| Method 4: GSADF (window size = 0.4) | 2012Q2 – 2022Q2 |

*Source: Author's calculations*

## 1.6 CONCLUSION: HOUSING MARKET AFTER COVID-19 PANDEMIC

After the Hong Kong housing bubble burst in 2022, it is expected that there will be a downward price correction in the near future. The decision of the Federal Reserve to raise the interest rate to 5.15% from 4 May 2023 exerts further downward pressure on the demand side of the housing market. Moreover, Hong Kong experienced an "emigration wave"[10], where a net outflow of 60,000 residents happened in 2022 (South China Morning Post, 16 February 2023). This suggests that some of the houses will be sold at the "fire-sale" prices in the secondary market. The following listed two challenges in the housing market:

- The downward trend of housing prices has a lot of implications for the developers' strategies. On the one hand, developers will be conservative in submitting land bids, thus the government revenue generated from the land sale will be substantially reduced. On the other hand, developers need to adjust the selling strategies of houses in the first-hand market. While Leung et al. (2020b) show that an oligopolistic structure existed among developers, this structure may be broken up during bad times. Having produced a downward price forecast, developers (especially those without deep pockets) may not "coordinate" with others and rush to sell their properties at a cheaper price for returning cash. It catches the attention of potential homebuyers and therefore absorbs huge funds from the market. The secondary housing market will be suppressed as a result.

[10] *See Chan (2022) for discussion.*

- Homebuyers need to evaluate the risk associated with the downward housing price movement. During the evolution of the housing bubble, homebuyers can make use of leverage to magnify the rate of return. However, in bad times, such a strategy will magnify the losses and even lead to bankruptcy. An illustrative example comes from the homebuyers of Grand Jete. These buyers make use of the “Stage Payment Plan” with high leverage to purchase the pre-sale units located in Phase 1, where they will pay the full amount upon housing completion. Unfortunately, when developers pre-sold the houses in Phase 2, they offered a huge discount (about 17% on average), exerting a downward price pressure on the Phase 1 housing units. Therefore, the bank valuation on Phase 1 property will be substantially reduced and it is questionable that the buyers are able to borrow enough money from banks to finish the transaction by the time of housing completion. If the buyers fail to do so, there is a risk that the developers will ask buyers for compensation.

To conclude, housing bubbles, where the price is significantly above the intrinsic value, were found in Hong Kong in the past three decades. This is mainly due to the low interest rate environment, limited supply for housing as well as the irrational exuberance and speculation. During the bubble period, investors were overconfident in purchasing the overvalued property, with a view of selling the property at an even higher price in the future. However, COVID-19 disrupts the landscape of the global market. This long-lasting pandemic has led to the burst of the recent housing bubble. The homebuyers who make use of high leverage to purchase the property at the price hike are susceptible to negative equity or bankruptcy. To better promote financial stability, government officials should have an eye on overall market conditions, in particular the mortgage default cases, and strengthen investor education. For further research, it is suggested to use big data analysis to create early warning signals of bubble creation and burst. This will provide policymakers an advance notice for taking proactive measures to stabilize the market.

## COMPETING INTERESTS

Author has declared that no competing interests exist.

## REFERENCES

Abraham, J. and Hendershott, O. (1996). Bubbles in Metropolitan Housing Markets. Journal of Housing Research, 7(2), 191-207.

Allen, D. and McAleer, M. (2021). Predicting COVID-19 Cases and Deaths in the USA from Tests and State Populations. Advances in Decision Sciences, 25(2), 1-27.

Andre, C., Caraiani, P., Calin, A. C. and Gupta, R. (2022). Can Monetary Policy Lean Against Housing Bubbles? Economic Modelling, 110, 105801.

Anundsen, A. K., Gerdrup, K., Hansen, F. and Kragh-Sorensen, K. (2016). Bubbles and Crises: The Roles of House Prices and Credit. Journal of Applied Econometrics, 31, 1291-1311.

Anundsen, A. K., Kivedal, B. K., Larsen, E. R. and Thorsrud, L. A. (2023). Behavioral Changes in the Housing Market Before and After the COVID-19. Journal of Housing Economics, 59.

Bai, Z. Hui, Y. C. and Wong, W. K. (2015). Internet Bubble Examination with Mean-Variance Ratio. Handbook of Financial Econometrics and Statistics, 1451-1465.

Bangura, M. and Lee, C. L. (2020). Housing Price Bubbles in Greater Sydney: Evidence from a Submarket Analysis. Housing Studies, 37(1), 1-36.

Barberis, N., Greenwood, R., Jin, L. and Shleifer, A. (2018). Extrapolation and Bubbles. Journal of Financial Economics, 129(2), 203-227.

Bourdeau-Brien, M. and Kryzanowski, L. (2020). Natural Disasters and Risk Aversion. Journal of Economic Behavior and Organization, 177, 818-835.

Brown, P., Daigneault, A. J., Tiernstrom, E. and Zou, W. (2018). Natural Disasters, Social Protection, and Risk Perceptions. World Development, 104, 310-325.

Case, K. E. and Shiller, R. J. (2003). Is There a Bubble in the Housing Market? Brookings Papers on Economic Activity, 2, 299-362.

Chan, A. K. W., Cheung, L. T. O., Chong, E. K. M., Lee, M. Y. K. and Wong, M. Y. H. (2022). Hong Kong's New Wave of Migration: Socio-Political Factors of Individuals' Intention to Emigrate. Comparative Migration Studies, 10, Article 49.

Chong, T. T. L., Li, X. and Yip, C. (2020). The Impact of COVID-19 on ASEAN. Economic and Political Studies, 166-185.

Cox, W. (2023). Demographia International Housing Affordability. Available at: http://www.demographia.com/dhi.pdf

Deng, Y., Han, C, Li, T. and Wang, Y. (2024). The Effectiveness and Consequences of the Government's Interventions for Hong Kong's Residential Housing Markets. Real Estate Economics.

Ding, H., Fan, H. and Lin, S. (2022). COVID-19, Firm Exposure and Firm Value: A Tale of Two Lockdowns. China Economic Review, 71.

Fama, E. F. (1965). The Behavior of Stock-Market Prices. Journal of Business, 38(1), 34-105.

Farmer, R. E. A. (2015). The Stock Market Crash Did Cause the Great Recession. Oxford Bulletin of Economics and Statistics, 77(5), 617-633.

Finance Chiefs Ease Negative Equity Fears. (2023, November 23). The Standard.

Foote, C. L., Gerardi, K. and Willen, P. S. (2008). Negative Equity and Foreclosure: Theory and Evidence. Journal of Urban Economics, 64(2), 234-245.

Foroni, C., Marcellino, M. and Stevanovic, D. (2020). Forecasting the COVID-19 Recession and Recovery: Lessons from the Financial Crisis. ECB Working Paper Series, No. 2468.

Garber, P. M. (2000). Famous First Bubbles: The Fundamentals of Early Manias. Cambridge: MIT Press.

Gharib, C., Mefteh-Wali, S. and Jabeur, S. B. (2021). The Bubble Contagion Effect of COVID-19 Outbreak: Evidence from Crude Oil and Gold Markets. Finance Research Letters, 38, 101703.

Glaeser, E. L and Nathanson, C. G. (2015). Housing Bubbles. Handbook of Regional and Urban Economics, 5, 701-751.
Gyourko, J. and Molly, R. (2015). Regulation and Housing Supply. Handbook of Regional and Urban Economics, 5, 1289-1337.
Homm, U. and Breitung J. (2012). Testing for Speculative Bubbles in Stock Market: A Comparison of Alternative Methods. Journal of Financial Econometrics, 10(1), 198-231.
Hong Kong's Population Drops for 3rd Straight Year, While City Posts Net Outflow of 60,000 Residents in 2022. (2023, February 16). South China Morning Post.
Hui, E. C. H., Liang, C., Wang, Z., Song, B. and Gu, Q. (2012). Real Estate Bubbles in China: A Tale of Two Cities. Construction Management and Economics, 30, 951-961.
Ji, X., Bu, N., Zheng, C., Xiao. H., Liu, C., Chen, X. and Wang, K. (2022). Stock Market Reaction to the COVID-19 Pandemic: An Event Study. Portuguese Economic Journal, 25, 1-20.
Jorda, O., Schularick, M. and Taylor, A. M. (2016). The Great Mortgaging: Housing Finance, Crises, and Business Cycles. Economic Policy, 31(85), 107-152.
Kung, J. J. and Wong, W. K. (2009). Profitability of Technical Analysis in the Singapore Stock Market Before and After the Asian Financial Crisis. Journal of Economic Integration, 24(1), 135-150.
Lai, R. N. and Van Order, R. A.(2020). A Tale of Two Countries: Comparing the US and Chinese Housing Markets. Journal of Real Estate Finance and Economics, 61(3), 505-547.
Lam, K. Liu, T. and Wong W. K. (2012). A New Pseudo-Bayesian Model with Implications for Financial Anomalies and Investors' Behavior. Journal of Behavioral Finance, 13(2), 93-107.
Lean, H. H., McAleer, M. and Wong W. K. (2015). Preferences of Risk-Averse and Risk-Seeking Investors for Oil Spot and Futures Before, During and After the Global Financial Crisis. International Review of Economics and Finance, 40, 204-216.
Leung, C. K. Y., Leung, T. C. and Tsang, K. P. (2015). Tax-driven Bunching of Housing Market Transactions: The Case of Hong Kong. International Real Estate Review, 18(4), 473-501.
Leung, C. K. Y., Leong, Y. C. F. and Wong, S. K. (2006). Housing Price Dispersion: An Empirical Investigation. Journal of Real Estate Finance and Economics, 32(3), 357-385.
Leung, C. K. Y., Ng, J. C. Y. and Tang, E, C. H. (2020a). Why is the Hong Kong Housing Market Unaffordable? Some Stylized Facts and Estimations. Quarterly Bulletin, Central Bank of the Republic of China (Taiwan), 42(1), 5-58.
Leung, C. K. Y., Ng, J. C. Y. and Tang, E, C. H. (2020b). What do we Know about Housing Supply? The Case of Hong Kong SAR. Economic and Political Studies, 8(1), 6-20.
Leung, C. K. Y. and Tang, E. C. H. (2011). Comparing Two Financial Crises: The Case of Hong Kong Housing Markets, in Global Housing Markets: Crises, Policies and Institutions, published by John Wiley & Sons.

Leung, C. K. Y. and Tang, E. C. H. (2023). The Dynamics of the House Price-to-Income Ratio: Theory and Evidence. Contemporary Economic Policy, 41(1), 61-78.

Li, S., Liu, J., Dong, J. and Li, X. (2021). 20 Years of Research on Real Estate Bubbles, Risk and Exuberance: A Bibliometric Analysis. Sustainability, 13(17), 9657.

Luangaram, P. and Thepmongkol, A. (2022). Loan-to-value Policy in a Bubble-creation Economy. Journal of Asian Economics, 79, 101433.

McAleer, M. Suen, J. and Wong, W. K. (2016). Profiteering from the Dot-com Bubble, Subprime Crisis and Asian Financial Crisis. The Japanese Economic Review, 67, 257-279.

Meen, G. (2002). The Time-series Behaviour of House Prices: A Transatlantic Divide. Journal of Housing Economics, 11, 1-23.

Munkh-Ulzii B. J., McAleer, M., Moslehpour, M. and Wong W. K. (2018). Confucius and Herding Behavior in the Stock Markets in China and Taiwan. Sustainability, 10(12), 4413.

Phillips, P. C. B., Wu, Y. and Yu, J. (2011). Explosive Behavior in the 1990s Nasdaq: When Did Exuberance Escalate Asset Values? International Economic Review, 52(1), 201-226.

Phillips, P. C. B., Shi, S. and Yu, J. (2015). Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500. International Economic Review, 56(4), 1043-1078.

Safi, S. K., Sanusi, O. I. and Tabash, M. I. (2022). Forecasting the Impact of COVID-19 Epidemic on China Exports Using Different Time Series Models. Advances in Decision Sciences, 26(1), 102-127.

Sun, Y., Bao, Q. and Lu, Z., 2021. Coronavirus (COVID-19) Outbreak, Investor Sentiment, and Medical Portfolio: Evidence from China, Hong Kong, Korea, Japan and US. Pacific-Basin Finance Journal, 65, 101463.

Taimouati, S., Wahbi, B. E. and Dakkon, M. (2022). Modeling COVID-19 Confirmed Cases Using a Hybrid Model. Advances in Decision Sciences, 26(1), 128-162.

Tang, Edward C. H. (2017). Real Estate Cycles and Housing Policies in Hong Kong, in Business Cycles: External / Internal Causes, Economic Implications and Consumer Misconceptions, published by Nova Science Publishers, 21-45.

Tang, E. C. H. (2021). Speculate a Lot. Pacific Economic Review, 26(1), 91-109.

Teng, H. J., Chang, C. O. and Chau, K. W. (2013). Housing Bubbles: A Tale of Two Cities. Habitat International, 39, 8-15.

Tuan, B. A., Pho, K. H., Pan, S. H. and Wong, W. K. (2022). Applications in Sciences in the Prevention of COVID-19. Advances in Decision Sciences, 26(4), 1-16.

Ulku, N., Ali, F., Saydumarov, S. and Ikizlerli, D. (2023). COVID Caused a Negative Bubble. Who Profited? Who Lost? How Stock Markets Changed? Pacific-Basin Finance Journal, 79.

Wong, S. K., Cheung, K. S., Deng, K. K. and Chau, K. W. (2021). Policy Responses to an Overheated Housing Market: Credit Tightening Versus Transaction Taxes. Journal of Asian Economics, 75, 101330.

Wong W. K. (2020). Review on Behavioral Economics and Behavioral Finance. Studies in Economics and Finance, 37(4), 625-672.

Yiu, M. S., Yu, J. and Jin, L. (2013). Detecting Bubbles in Hong Kong Residential Property Market. Journal of Asian Economics, 28, 115-124.

Zhu, Z. Bai, Z. Vieito J. P. and Wong, W. K. (2019). The Impact of the Global Financial Crisis on the Efficiency and Performance of Latin American Stock Markets. Estudios de Economia, 46(1), 5-30

**Biography of author(s)**

**Edward Chi-Ho Tang**
Hong Kong Shue Yan University, Hong Kong.

He is an Associate Professor at the Department of Economics and Finance of Hong Kong Shue Yan University. He obtained PhD at the City University of Hong Kong. His research interests include real estate economics and empirical asset pricing. He has published many articles in Contemporary Economic Policy, International Real Estate Review, Journal of Macroeconomics, Pacific Economic Review, Regional Science and Urban Economics, and Singapore Economic Review. Also, he was invited to be one of the guest editors of the Singapore Economic Review (Special Issue on Asian Consumer Economics). In 2020-2021, he was the Principal Investigator of a project funded under RGC's Faculty Development Scheme.



**Peer-Review History:**
This chapter was reviewed by following the Advanced Open Peer Review policy. This chapter was thoroughly checked to prevent plagiarism. As per editorial policy, a minimum of two peer-reviewers reviewed the manuscript. After review and revision of the manuscript, the Book Editor approved the manuscript for final publication. Peer review comments, comments of the editor(s), etc. are available here: https://peerreviewarchive.com/review-history/12411F-CH1

***Chapter 2***
Print ISBN: 978-81-973195-8-7, eBook ISBN: 978-81-973195-0-1

# Study on Bubbles Solutions of the Cagan Model

**Kai-Yin Woo [a++*]**

DOI: 10.9734/bpi/mono/978-81-973195-8-7/CH2

**Peer-Review History:**
This chapter was reviewed by following the Advanced Open Peer Review policy. This chapter was thoroughly checked to prevent plagiarism. As per editorial policy, a minimum of two peer-reviewers reviewed the manuscript. After review and revision of the manuscript, the Book Editor approved the manuscript for final publication. Peer review comments, comments of the editor(s), etc. are available here: https://peerreviewarchive.com/review-history/12411F-CH2

## ABSTRACT

The purpose of this paper is to study bubbles solutions to the Cagan hyperinflation models under rational expectations. Both the price and exchange rate bubbles are considered. Specifications of the Cagan model under rational expectations will be briefly described, in which the price and exchange rate series are expressed in first-order linear difference equations. The particular and the homogenous solutions to the Cagan model can then be derived. The particular or fundamental solution characterizes a unique dynamic movement of an underlying fundamental process. Several representations of the fundamental solution will be explored. The homogenous or bubble solution is non-unique in a rational expectations framework. Some examples of bubble solution with different dynamic properties are specified. Also, examples of bursting bubble specifications will be illustrated. It is concluded that the problems of multiple solutions make indirect tests more attractive than direct tests for bubble detection. In addition, the general solution, which is just the sum of particular and homogenous solutions, will be discussed. Hence, the bubble paths are characterized as any deviations of the general solution from the fundamental solution when the model is specified correctly.

*Keywords: Cagan model; price bubbles; exchange rate bubbles; particular solution; homogenous solution.*

**JEL classifications:** C6, F4.

*[a] Department of Economics and Finance, Hong Kong Shue Yan University, Hong Kong.*
*[++] Associate Professor;*
**Corresponding author: E-mail: kywoo@hksyu.edu;*

## 2.1 INTRODUCTION

Price bubbles are defined as explosive processes of asset prices generated by self-fulfilling expectations independently of market fundamentals. The existence of bubbles represents a possible explanation for the deviation of asset prices from the underlying fundamentals. There are many historical examples of incidents that could be considered from the evidence as being self-fulfilling bubbles. Famous classic cases include the tulip mania in the Netherlands from 1634 to 1637, 'the Mississippi bubble' in France in 1719-1720 and the contemporaneous and related 'South Sea bubbles' in Britain (Garber, 1989 and 1990). In addition, the US stock market crashes of 1929, 1987 and 2000, the Asian stock market slump of 1997, as well as the Japanese property market crash in the 1990s, are usually deemed as recent examples of bubble bursts. Keynes (1936) considered that the stock prices in the 1920s might not be governed by an objective view of fundamentals but by "what average opinion expects the average opinion to be". The study of bubbles has attracted much research interest because bubble bursts will normally have negative wealth effects and create economic confusion (Blanchard and Watson, 1982). According to Kindleberger (1987), a bubble is defined loosely as a sharp rise in the price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers, who are generally speculators interested in profits from trading in the asset rather than its use of earning capacity; the rise is usually followed by a reversal of expectations and a subsequent sharp decline in price often resulting in financial crisis.

In the literature, general equilibrium arguments can be found about the theoretical restrictions concerning the existence of bubbles and the effects of bubbles on the economy. For instance, Tirole (1982) considers that rational bubbles are ruled out when there exists a finite number of agents in the market. If the number of agents is infinite, bubble existence will become possible (Tirole, 1985; Weil, 1989). On the other hand, within a monetary framework, Obstfeld and Rogoff (1983) assert that price bubbles can be ruled out during hyperinflationary episodes if the government guarantees a probable minimal redemption value for the currency in units of capital.

Although the study on bubbles has largely focused on capital markets, I contend that the investigation of inflationary bubbles is equally important. Since the choice of an appropriate policy to reduce the inflation rate may very much depend on the true nature of the underlying process generating the inflation, the existence of inflationary bubbles has far-reaching policy implications. If inflationary bubbles are not present in the observed price series, then it is only necessary to take control of the market fundamentals, by such means as the restrictive control of money supply growth and the reduction of fiscal deficits. If, however, this inflation has a stubborn self-sustaining momentum and is thus being driven by a bubble phenomenon, then positive action will be required to work on the expectation mechanism to shock expectations off the speculative bubble path (Funke et al., 1994). For instance, it would require the government to commit itself to a change in its policies for controlling fiscal deficits and money growth in a way that is sufficiently binding and convincing for them to be widely believed. Further, since bubbles are associated with self-fulfilling prophecies, it is reasonable to deduce that if bubbles do occur in the data, they are more likely to be observed when the expected future market price is an important factor determining the current market price level. During hyperinflation, expectation plays a dominant role in the determination of the asset price. Hence, it is believed that hyperinflationary episodes provide fascinating environments for the empirical study of bubbles (Flood and Garber, 1980). The classic examples include the inter-war European hyperinflations of Germany, Hungary and Poland. Sargent (1982) provides a detailed description of how the hyperinflation in these countries was stopped. It has been found that the government authorities stopped inflation by announcing a binding and credible policy regime change and at the same time taking control of market fundamentals. Thus, the resulting control of inflation cannot explain fully the true nature of the hyperinflation that occurred.

It is also to be noted that a floating exchange rate system was first implemented in European countries during the 1920s following World War I. According to Okina (1984), if price bubbles occur and the purchasing power parity (PPP) is not violated, bubbles in the nominal exchange rate will also exist and are reflected in the form of the price bubbles. The country’s external competitiveness, therefore, would not be adversely affected. On the other hand, when price bubbles are not present, but exchange rate bubbles do exist, the nominal exchange rate bubbles are represented by an explosive deviation from PPP, and real exchange rate bubbles will exist as well. With the ups and pops of real exchange rate fluctuations, the export sectors will suffer serious consequences and will not recover quickly even when the bubbles finally burst.

The study on bubbles is undertaken in the Cagan model during hyperinflationary episodes (Taylor, 1991). The purpose of this paper is to study bubbles solutions to the Cagan hyperinflation models under rational expectations. Both the price and exchange rate bubbles are considered. I will briefly describe the specifications of the Cagan model under rational expectations in which the price and exchange rate series are expressed in first-order linear difference equations. The particular and the homogenous solutions to the Cagan model can then be derived. The particular or fundamental solution characterizes a unique dynamic movement of an underlying fundamental process. Several representations of the fundamental solution will be explored. The homogenous or bubble solution is non-unique in a rational expectations framework. I attempt to specify some examples of bubble solutions with different dynamic properties. Also, examples of bursting bubble specifications will be illustrated. It is concluded that the problems of multiple solutions make indirect tests more attractive than direct tests for bubble detection. In addition, the general solution, which is just the sum of particular and homogenous solutions, will be discussed. Hence, the bubble paths are characterized as any deviations of the general solution from the fundamental solution when the model is specified correctly.

The remainder of the paper proceeds as follows. The specifications of the Cagan hyperinflation models, with particular, homogenous and general solutions, will be explored in Section 2. A summary is offered in the final section. Proofs of some equations are shown in the appendices.

## 2.2 CAGAN MODEL

### 2.2.1 Model Specifications

Money balances are held as a reserve of purchasing power for contingencies. The desired real money balances depend upon several variables including real wealth, real income, and the expected opportunity cost of holding money. The expected cost of holding money refers to the difference between the expected monetary return on holding cash balance and on substitutes of local currency. The money return on cash balance is negligible and is usually assumed to be zero. Therefore, to the extent that money is held as a substitute for financial assets, the expected cost of holding money includes the expected interest rate and capital gain yield of holding those financial assets. To the extent that money is held as substitutes for non-perishable consumers' goods, the expected cost of holding money is the expected rate of depreciation in the real value of money, or equivalently, the rate of inflation. According to Cagan (1956), hyperinflation refers to the rise in prices at a rate at least equal to 50% per month and only the expected inflation rate accounts for the drastic fluctuations in real cash balances during hyperinflation, with all other variables being considered to have minor effects on desired cash balance. Cagan (1956) assumes the expectation mechanism to be adaptive. Sargent and Wallace (1973), Sargent (1977) and Salemi and Sargent (1979), however, introduce the rational expectation hypothesis of Muth (1961) to the Cagan model. Mathematically, the linear form of the Cagan model under rational expectations and instantaneous clearing in the money market is given as:

$$M_t - \pi_{1,t} = \alpha_1 + \beta_1 E_t(\Delta\pi_{t+1}) \ + \ u_{1,t} \tag{1}$$

where $M_t$ is the natural logarithm of the money stock at time t, $\pi_{1,t}$ is the natural logarithm of the price level, $E_t(.)$ denotes the mathematical expectations operator conditional on information set $\Omega t$, $\alpha_1$ is a constant, $\beta_1$ is the semi-elasticity of real money demand with respect to the expected inflation rate and $u_{1,t}$ refers to a money demand disturbance term representing all deviations from the exact Cagan model under rational expectations such as demand velocity shocks and all other omitted real variables. Theoretically, the value of $\beta_1$ should be negative because money holders will substitute consumers' goods for money when the real value of money is expected to fall or the expected inflation rate rises.

Since local currency loses its value very rapidly during hyperinflation, foreign currency balances are often held in order to perform the functions of a medium of exchange and a store of value. Even if foreign currencies are held merely as a store of value, they are often converted back into domestic money and then goods at a later time. Hence, the substitution between domestic money and goods can occur, directly or indirectly, via foreign currencies (Moosa, 1999). Such phenomenon of currency substitution is documented in Sargent (1982) for the inter-war European hyperinflations. In light of this, it is appropriate to replace the future inflation rate in (1) with the expected depreciation rate of domestic currency to represent the cost of holding domestic money balance. If I further assume that the PPP relationship holds and that all the foreign money demand determinants, for example, foreign interest rates and income levels, are assumed to be constant, the real money balance represented by (1) can be alternatively expressed as:

$$M_t - \pi_{2,t} = \alpha_2 + \beta_2 E_t(\Delta\pi_{2,t+1}) + u_{2,t} \tag{2}$$

where $\pi_{2,t}$ is the natural logarithm of the exchange rate measured as the value of domestic currency per unit of foreign currency, $\Delta\pi_{2,t+1}$ represents the exchange rate change at time $t$+1, $\alpha_2$ is an intercept, $\beta_2$ is the semi-elasticity of currency substitution between domestic and foreign currency and $u_{2,t}$ is a measure of model noise from the linear exact Cagan model under rational expectations that include all the domestic and foreign money demand shocks and omitted real-side determinants.

Re-arranging (1) and (2) in terms of $\pi_{j,t}$ ($j$ = 1, 2) gives:

$$\pi_{j,t} = \frac{\alpha_j}{\beta_j - 1} - \frac{M_t}{\beta_j - 1} + \frac{\beta_j}{\beta_j - 1} E_t(\pi_{j,t+1}) + \frac{u_{j,t}}{\beta_j - 1} \qquad j = 1, 2. \tag{3}$$

Eq. (3) is expressed as a first-order dynamic linear difference equation with rational expectations. The future expectation and the current variables are determined simultaneously. The general solution of (3) is the sum of a particular solution and a homogenous solution.

## 2.2.2 Particular Solution

For sake of notional simplicity, I eliminate the subscript $j$ in subsequent equations. By recursively substituting forward for $E_t(\pi_{t+1+i})$ and using the law of iterated expectations, I obtain:

$$\pi_t = -\alpha + \frac{1}{1-\beta}\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(M_{t+i} - u_{t+i}) + \lim_{i\to\infty}\left(\frac{\beta}{\beta-1}\right)^{i+1} E_t(\pi_{t+1+i}) \tag{4}$$

When $\left|\frac{\beta}{\beta-1}\right| < 1$, the transversality condition:

$$\lim_{i\to\infty}\left(\frac{\beta}{\beta-1}\right)^{i+1} E_t(\pi_{t+1+i}) = 0, \tag{5}$$

is then satisfied. Under this circumstance, the solution of $\pi_t$ is given by:

$$\pi_t^f = -\alpha + \frac{1}{1-\beta}\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(M_{t+i} - u_{t+i}) \tag{6}$$

The expression of $\pi_t^f$ represents a forward-looking particular solution or the fundamental solution to the Cagan models (1) and (2) under rational expectations, which is determined by the present discounted value of expected levels of the market fundamentals, $(M_{t+i} - u_{t+i})$, for all i ≧ 0. If the expectation of $(M_t - u_t)$ grows at a constant rate g, the infinite sum, $\pi_t^f$, will converge when (1+g) < $\frac{\beta-1}{\beta}$ or g < $|\frac{1}{\beta}|$.

By assuming special stochastic processes for the sequence of $M_t$ and $u_t$, $\pi_t^f$ can be written in explicit manners. Let's define $X_t$ as $\frac{1}{1-\beta}M_t$ or $\frac{-1}{1-\beta}u_t$. Gourieroux *et al.* (1982) considers the ARMA solutions of the Cagan model. Assume that $|\frac{\beta}{\beta-1}|<1$ and $X_t$ admits an ARMA (p, q) representation, that is, $\Psi(L)X_t$ = $\theta(L)\eta_t$, where $L$ is a lag operator, $\Psi(L) = 1-\Psi_1 L - ... - \Psi_p L^p$, $\theta(L)$ = $1+\theta_1 L + ... + \theta_q L^q$ and $\eta_t$ is a white noise. Then, the present discounted value of $X_t$, $\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(X_{t+i})$ can be explicitly written as a unique stationary ARMA solution:

$$\{\frac{1}{(L-b)}[L - \frac{b\theta(b)\Psi(L)}{\Psi(b)\theta(L)}]\}X_t, \tag{7}$$

where ***b*** is defined as $\frac{\beta}{\beta-1}$.

However, many economic variables exhibit non-stationarity. If $X_t$ follows random walk with drift and linear time trend, that is, $\Delta X_t$ = $\mu + \omega t + \eta_t$, then the present discounted value of $X_t$ is written as (see Appendix 1):

$$X_t + \beta(\beta-1)\{\mu + \omega[(1-\beta)+t]\}, \tag{8}$$

During hyperinflation, the economic variables are likely to contain double unit roots (Haldrup, 1998). I then consider the case of double unit roots with drift and polynomial time trend. Assume that $\Delta^2 X_t = \mu + \omega_1 t + \omega_2 t^2 + \eta_t$. The infinite sum of $X_t$ will be represented as (see Appendix 2):

$$(1-\beta)X_t + (1-\beta)^2 \Delta X_t - \beta(1-\beta)^2 \{\mu + \omega_1(1-\beta) + \omega_2[(1-\beta)^2 - \beta(1-\beta) + t^2] + t[\omega_1 + 2\omega_2(1-\beta)]\}, \quad (9)$$

Since the fundamentals, $M_t$ and $u_t$, may be represented by different stochastic processes, for instance, $M_t$ is usually I(2) and $u_t$ is either I(1) or I(0), the explicit representation of the fundamental solution, $\pi_t^f$, will be written as a combination of Eqs.(7), (8) and (9). The conditions for the particular solutions of the Cagan model to be unique are documented in Broze and Szafarz (1991) and Broze *et al.* (1995).

### 2.2.3 Homogenous Solution

The homogenous solution of (3) denoted by $\pi_t^h$ is equal to the general solution of the homogenous counterpart as follows:

$$\pi_t^h = b^{1+i} E_t(\pi_{t+1+i}^h), \quad i \geqq 0 \quad (10)$$

Multiplying both sides of (10) by $b^t$ obtains:

$$b^t \pi_t^h = b^{t+1+i} E_t(\pi_{t+1+i}^h), \quad (11)$$

Gourieroux *et al.* (1982) derive the homogenous solution, $\pi_t^h$, by using the martingale process. Let's define $m_t$ as $b^t \pi_t^h$, and the stochastic process of $m_t$ satisfies the martingale property such that $E_t(m_t) = m_t$, and $E_t(m_{t+i}) = m_t$, for all i > 0. The homogenous solution, $\pi_t^h$, is represented in terms of the martingale process:

$$\pi_t^h = \frac{m_t}{b^t} \quad (12)$$

Any arbitrary martingale process, $m_t$, can be considered as a component of $\pi_t^h$. It implies the existence of multiple solutions for the Cagan models under rational expectations. Since $E_t(\pi_{t+1}^h) = E_t(\frac{m_{t+1}}{b^{t+1}}) = \frac{1}{b}(\frac{m_t}{b^t})$ where $|b| < 1$, the stochastic process of $\pi_t^h$ follows a sub-martingale such that:

$$E_t(\pi_{t+1}^h) = \frac{\pi_t^h}{b} > \pi_t^h . \tag{13}$$

Therefore, $\pi_t^h$ satisfies a bubble process that explodes in expected value, and it can be interpreted as a bubble solution, $B_t$ . Also, when (11) and (12) are substituted into the transversality condition of (5), it implies $E_t(m_{t+1+i}) = m_t = 0,$ for i$\rightarrow\infty$. Consequently, the transversality condition of (5) implies the nonexistence of $B_t$ .

The stochastic unit root process suggested by Granger and Swanson (1997) can be generalized to the martingale process, $m_t$ :

$$m_t = q_t m_{t-1} + \omega_t , \tag{14}$$

where $E_t(q_{t+1}) = E_{t-1}(q_t) = 1, \quad E_t(\omega_{t+i}) = 0,$ for all i > 0.

Suppose that $x_t$ ~ N $(\mu_x, \sigma_x^2).$ For an arbitrary $\lambda$ , the moment-generating function of a normally distributed variable, $x_t$ , is given by $E$ (exp( $\lambda x_t$ )) = exp( $\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2$). Hence, $q_t$ is represented by $\exp[\lambda x_t - (\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2)]$. Dividing (14) by $b^t$ yields the following general bubble specification:

$$B_t = \frac{(q_t B_{t-1})}{b} + \frac{\omega_t}{b^t} \tag{15a}$$

$$= \exp[\lambda x_t - (\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + \ln b)] \; B_{t-1} + \frac{\omega_t}{b^t} \tag{15b}$$

By restricting underlying parameters of bubble process given by (15b) such as $\lambda$ and $\mu_x$ , there are different theoretical bubble specifications with particular stochastic properties to be derived (Salge, 1997). I illustrate them with further modifications and refinements.

#### 2.2.3.1 $\lambda$ = 0

Let's first assume that $\lambda$ = 0, the resulting bubble process will be obtained as follows:

$$B_t = \frac{B_{t-1}}{b} + \frac{\omega_t}{b^t} = \frac{B_0}{b^t} + \frac{\sum_{i=1}\omega_{t=i}}{b^t} \tag{16}$$

Since the above bubble process is driven by time only, it is known as pure time-driven bubble process. As ***t***$\rightarrow\infty$, the time-driven bubble must converge toward infinity with $|\, b\,|$ < 1 and its dynamics must then be asymptotically unstable. In particular, if $m_t$ is a constant, the sequence of $\omega_t$ in (14) and (16) will become zero. Consequently, $B_t$ is represented by $\frac{B_0}{b^t}$ only, which is known as the deterministic bubble.

**2.2.3.2 $\lambda \neq 0$ and $\mu_x + \frac{1}{2}\lambda\, \sigma_x^2 = 0$**

If $\lambda \neq 0$ and $\mu_x + \frac{1}{2}\lambda\, \sigma_x^2 = 0$, it then implies $\mu_x \neq 0$ because $\lambda = \frac{-2\mu_x}{\sigma_x^2} \neq 0$. The bubble process can be specified as:

$$B_t = \exp(\frac{-2\mu_x}{\sigma_x^2} x_t - \ln b)\ B_{t-1} + \frac{\omega_t}{b^t} \tag{17a}$$

Suppose that $x_t = w_t - w_{t-1} = \mu_x + \varepsilon_{xt}$, where $\varepsilon_{xt} \sim N(0, \sigma_x^2)$. Replacing $x_t$ by $w_t - w_{t-1}$, substituting one period forward for $B_{t+1}$ and re-arranging yield (see Appendix 3):

$$B_t = \exp[\frac{-2\mu_x}{\sigma_x^2} w_t - (\ln b)t] \tag{17b}$$

where $\ln b < 0$ since $b < 1$ or $\beta < 0$.

Assume that $w_t$ represents a vector of underlying I(1) fundamental variables in the model. The stochastic bubble process of (17b) thus depends upon both time and the underlying fundamental process. Further, by recursively forward substitution, $w_t = w_0 + \mu_x t + \sum_{i=1}^{t} \varepsilon_{xt-i}$, the bubble process given by (17b) can be alternatively written as:

$$B_t = \exp\{\frac{-2\mu_x}{\sigma_x^2} w_0 - [\frac{2\mu_x}{\sigma_x^2}(\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t}) + \ln b]t\} \tag{17c}$$

Given that $\frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t} \rightarrow 0$ as $t \rightarrow \infty$, $B_t$ will converge toward zero when $\frac{-2\mu_x}{\sigma_x^2} \mu_x < \ln b < 0$. As a result, the dynamics of $B_t$ is asymptotically stable. The divergent bubble process driven by the time component, $\exp[-(\ln b)t]$, would be somehow offset by the fundamental component, $\exp[\frac{-2\mu_x}{\sigma_x^2} \mu_x]$, to a certain degree. Hence, the inclusion of the fundamental-dependent component may help stabilize bubble dynamics and exhibit more dynamic properties of the bubble process (Ikeda and Shibata, 1992 and 1995).

One special case is that $\mu_x + \frac{1}{2}\lambda\ \sigma_x^2$ = 0 but $\lambda$ is restricted to be 1, $\mu_x = -\frac{\sigma_x^2}{2}$ < 0 and then the bubble process of (17a) and (17c) will be simplified to be:

$$B_t = \exp(x_t - \ln b)\ B_{t-1} + \frac{\omega_t}{b^t} \qquad (18a)$$

$$= \exp\{w_0 + (\mu_x + \frac{\sum_{i=1}^{t}\varepsilon_{xt-i}}{t} - \ln b)t\} \qquad (18b)$$

Similarly, the asymptotic dynamics of bubble process given by (18b) depend on the sign of $(\mu_x - \ln b)$. If $\mu_x < \ln b < 0$ or $\exp(\mu_x) < b < 1$, $B_t$ will converge toward zero and is then asymptotically stable.

**2.2.3.3 $\lambda$ ≠0 and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + K)$ = 0**

Let $\ln b$ = (K + H) < 0, where K and H are arbitrary constants. Assume that $\lambda$ ≠0 and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + K)$ = 0 with $\lambda_1$ and $\lambda_2$ being the two characteristic roots. Hence,

$$B_t = \exp(\lambda_1 w_t - \mathrm{H}\,t) \qquad (19a)$$

$$\text{or} \quad B_t = \exp(\lambda_2 w_t - \mathrm{H}\,t) \qquad (19b)$$

$$\text{where } \lambda_1 = \frac{-\mu_x + [\mu_x^2 - 4(\frac{1}{2})\sigma_x^2\mathrm{K}]^{1/2}}{2(\frac{1}{2})\sigma_x^2} = \frac{-\mu_x + (\mu_x^2 - 2\sigma_x^2\mathrm{K})^{1/2}}{\sigma_x^2} \qquad (20a)$$

$$\lambda_2 = \frac{-\mu_x - (\mu_x^2 - 2\sigma_x^2\mathrm{K})^{1/2}}{\sigma_x^2} \qquad (20b)$$

I first consider the case of $\mu_x \neq 0$. While $\mu_x \neq 0$ and $(\mu_x^2 - 2\sigma_x^2\mathrm{K})$ = 0, then, K = $\frac{\mu_x^2}{2\sigma_x^2}$ > 0, H must be negative. From (20a) and (20b), $\lambda_1 = \lambda_2 = -\frac{\mu_x}{\sigma_x^2}$. The bubble process of (19a) and (19b) will be written as:

$$B_t = \exp(-\frac{\mu_x}{\sigma_x^2} w_t - \mathrm{H}t) \qquad (21a)$$

$$= \exp\{-\frac{\mu_x}{\sigma_x^2} w_0 - [\frac{\mu_x}{\sigma_x^2}(\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t}) + \text{H}]t\}, \quad (21b)$$

When $-\frac{\mu_x^2}{\sigma_x^2}$ < H, which implies $\exp(-\frac{\mu_x^2}{2\sigma_x^2}) < b$, the bubble process will converge towards zero asymptotically.

On the other hand, when $(\mu_x^2 - 2\sigma_x^2\text{K})$ ≠0, then it can be seen that $\lambda_1 \neq \lambda_2$. The bubble process of (19a) or (19b) or any linear combination of them still satisfies the sub-martingale process of (13). Let's define $A_1$ and $A_2$ as two arbitrary constants. The linear combination of bubble process (19a) and (19b) is given as:

$$B_t = \text{A}_1 \exp(\lambda_1 w_t - \text{H}t) + \text{A}_2 \exp(\lambda_2 w_t - \text{H}t) \quad (22a)$$

$$= \text{A}_1 \exp(\lambda_1 w_0 + [\lambda_1(\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{\text{xt-i}}}{t})\text{-H}]t) + \text{A}_2 \exp(\lambda_2 w_0 + [\lambda_2(\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{\text{xt-i}}}{t})\text{-H}]t) \quad (22b)$$

The bubble solution (22a) can satisfy the sub-martingale property (see Appendix 4).

While $(\mu_x^2 - 2\sigma_x^2\text{K})$ > 0, or K < $\frac{\mu_x^2}{2\sigma_x^2}$, then $\lambda_1 > \lambda_2$ and the $\lambda$ values are real numbers. The stochastic stability of bubbles specified by (22b) depends upon whether $\lambda_i \mu_x < \text{H}$ or $\exp(\lambda_i \mu_x + \text{K}) < b$ for all i = 1,2.

Moreover, if $(\mu_x^2 - 2\sigma_x^2\text{K})$ < 0, or K > $\frac{\mu_x^2}{2\sigma_x^2}$ > 0, then H must be negative and the $\lambda$ values contain imaginary numbers:

$$\lambda_1 = \frac{-\mu_x + [(2\sigma_x^2\text{K-}\mu_x)^{1/2}]i}{\sigma_x^2}, \quad (23a)$$

$$\lambda_2 = \frac{-\mu_x - [(2\sigma_x^2\text{K-}\mu_x)^{1/2}]i}{\sigma_x^2}, \quad (23b)$$

where ***i*** is an imaginary number, $\sqrt{-1}$. Let's define $h_1 = \frac{-\mu_x}{\sigma_x^2}$, and $h_2 = \frac{[(2\sigma_x^2\text{K-}\mu_x)^{1/2}]}{\sigma_x^2}$, so that $\lambda_1, \lambda_2 = h_1 \pm h_2$ ***i***. The bubble process is specified as (see Appendix 5):

$$B_t = \exp(\text{h}_1 w_t\text{-H}t)[\text{A}_3 \cos(\text{h}_2 w_t) + \text{A}_4 \sin(\text{h}_2 w_t)] \quad (24a)$$

$$= \exp\{h_1 w_0 + h_1(\mu_x + \frac{\sum_{i=1}^{t}\varepsilon_{xt-i}}{t})t - Ht\}[A_3\cos(h_2 w_t) + A_4\sin(h_2 w_t)] \qquad (24b)$$

Under this circumstance, the bubble process of (24a) and (24b) can exhibit cyclical patterns. While $\frac{-\mu_x^2}{\sigma_x^2}$ < H < 0, or $\exp(\frac{-\mu_x^2}{\sigma_x^2} + K)$ < ***b***, the cyclical dynamics of $B_t$ is asymptotically damped.

Now, I consider the case of $\mu_x = 0$. When $\mu_x = 0$, then K must be negative since $(\frac{1}{2}\lambda^2\sigma_x^2 + K)$ = 0. Also, $\lambda_1 = \frac{(-2\sigma_x^2 K)^{1/2}}{\sigma_x^2}$ = $\frac{(-2K)^{1/2}}{\sigma_x}$ > 0, $\lambda_2 = -\frac{(-2K)^{1/2}}{\sigma_x}$ < 0. The bubble process will be specified as:

$$B_t = A_1\exp[\frac{(-2K)^{1/2}}{\sigma_x}w_t - Ht] + A_2\exp[-\frac{(-2K)^{1/2}}{\sigma_x}w_t - Ht] \qquad (25a)$$

$$B_t = A_1\exp(\lambda_1 w_0 + [\lambda_1(\frac{\sum_{i=1}^{t}\varepsilon_{\text{xt-i}}}{t})\text{-H}]t) + A_2\exp(\lambda_2 w_0 + [\lambda_2(\frac{\sum_{i=1}^{t}\varepsilon_{\text{xt-i}}}{t})\text{-H}]t) \qquad (25b)$$

On condition that H > 0, the bubble process (25b) will converge towards zero as ***t***$\rightarrow\infty$.

**2.2.3.4 $\lambda$ ≠0 and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + \ln b)$ = 0**

Suppose that $\lambda$ ≠0 and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + \ln b)$ = 0, the bubble process will be purely driven by a fundamental process:

$$B_t = \exp(\lambda_1 w_t) \qquad (26a)$$

or

$$B_t = \exp(\lambda_2 w_t) \qquad (26b)$$

where

$$\lambda_1 = \frac{-\mu_x + (\mu_x^2 - 2\sigma_x^2 \ln b)^{1/2}}{\sigma_x^2} \qquad (27a)$$

$$\lambda_2 = \frac{-\mu_x - (\mu_x^2 - 2\sigma_x^2 \ln b)^{1/2}}{\sigma_x^2} \qquad (27b)$$

Given the fact that ln ***b*** < 0, when $\mu_x \neq 0$, it is impossible for $(\mu_x^2 - 2\sigma_x^2 \ln b)$ = 0 and $(\mu_x^2 - 2\sigma_x^2 \ln b)$ **<** 0, which imply that ln ***b*** = $\frac{\mu_x^2}{2\sigma_x^2}$ > 0 and ln ***b*** > $\frac{\mu_x^2}{2\sigma_x^2}$ > 0 respectively. The only possible case is given by $(\mu_x^2 - 2\sigma_x^2 \ln b) > 0$, or ln ***b*** < 0 < $\frac{\mu_x^2}{2\sigma_x^2}$. Then, $\lambda_1 > \lambda_2$ and the $\lambda$ values are real numbers. The specification of the bubble process will be written as:

$$B_t = A_1 \exp(\lambda_1 w_t) + A_2 \exp(\lambda_2 w_t) \quad (28a)$$

$$= A_1 \exp(\lambda_1 w_0 + [\lambda_1(\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t})]t) + A_2 \exp(\lambda_2 w_0 + [\lambda_2(\mu_x + \frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t})]t) \quad (28b)$$

The stochastic stability of bubbles specified by (28b) depends upon whether $\lambda_i \mu_x < 0$ for all i = 1,2.

In case of $\mu_x = 0$, then $\lambda_1 = \frac{(-2\sigma_x^2 \ln b)^{1/2}}{\sigma_x^2}$ **=** $\frac{(-2\ln b)^{1/2}}{\sigma_x}$ **>** 0 and $\lambda_2 = \frac{-(-2 \ln b)^{1/2}}{\sigma_x}$ **<** 0 since ln ***b*** must be negative**.** The bubble process is expressed as:

$$B_t = A_1 \exp[\frac{(-2\ln b)^{1/2}}{\sigma_x} w_t] + A_2 \exp[-\frac{(-2\ln b)^{1/2}}{\sigma_x} w_t] \quad (29a)$$

$$= A_1 \exp\{\frac{(-2\ln b)^{1/2}}{\sigma_x}[w_0 + (\frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t})t]\} + A_2 \exp\{-\frac{(-2\ln b)^{1/2}}{\sigma_x}[w_0 + (\frac{\sum_{i=1}^{t} \varepsilon_{xt-i}}{t})t]\} \quad (29b)$$

The bubble process of (29b) must exhibit stable dynamics as ***t***$\rightarrow \infty$.

From the above, although all bubble processes are derived to explode in expected values, they may converge towards zero as ***t***$\rightarrow \infty$ under certain restrictions on parameters. The different examples of bubble specifications are summarized in Table 2.1. The asymptotic stability of bubble process leads to difficulties in bubble testing. Nevertheless, the numbers of observations are usually not large during hyperinflationary episodes and consequently, such difficulties may not be so serious, although it may create serious problems of bubble detection in financial markets with long data horizons.

Other than the asymptotic dynamics of bubble, the bursting properties are the main issues about the theoretical specifications of bubble solution. The sub-martingale property of bubble process (13) can be further modified by the inclusion of a probability that a bubble continues to grow ($0 \leqq \Pi \leqq 1$):

$$E_t(B_{t+1}) = \Pi \; E_t(B_{t+1} \mid G) + (1-\Pi)\, E_t(B_{t+1} \mid C) = \frac{B_t}{b} \quad (30)$$

where $E_t(B_{t+1} \mid G)$ and $E_t(B_{t+1} \mid C)$ refer to the expected values of $B_{t+1}$ given the regimes of bubble growth (G) and bubble collapse (C) respectively.

**Table 2.1. Summary for different theoretical bubble specifications**

| Parameter restrictions | Equations | Conditions for dynamic stability |
|---|---|---|
| $\lambda = 0$ | 16 | None |
| $\mu_x + \frac{1}{2}\lambda\,\sigma_x^2 = 0$ | | |
| $\mu_x \neq 0$ and $\lambda \neq 0$ | 17a, b, c | $\frac{-2\mu_x}{\sigma_x^2}\mu_x < \ln b < 0.$ |
| $\mu_x = -\frac{\sigma_x^2}{2} < 0$ and $\lambda = 1$ | 18a, b | $\mu_x < \ln b < 0$ |
| $\lambda \neq 0$, and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + K) = 0$, where $\ln b = (K + H) < 0$ | | |
| $\mu_x \neq 0$ and $\lambda_1 = \lambda_2$ | 21a, b | $-\frac{\mu_x^2}{\sigma_x^2} < H$ |
| $\mu_x \neq 0$ and $\lambda_1 \neq \lambda_2$ | 22a,b | $\lambda_i\mu_x < H$ |
| $\mu_x \neq 0$ and $\lambda_1, \lambda_2 = h_1 \pm h_2 \boldsymbol{i}$ | 24a,b | $\frac{-\mu_x^2}{\sigma_x^2} < H < 0$ |
| $\mu_x = 0$ and $\lambda_1 \neq \lambda_2$ | 25a,b | $H > 0$ |
| $\lambda \neq 0$ and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + \ln b) = 0$ | | |
| $\mu_x \neq 0$ and $\lambda_1 \neq \lambda_2$ | 28a,b | $\lambda_i\mu_x < 0$ |
| $\mu_x = 0$ and $\lambda_1 \neq \lambda_2$ | 29a,b | Must be asymptotically stable |

One particular example of a bubble process that is satisfied with the above bursting bubble specification (30) is given as:

$B_{t+1} = \exp\{\lambda x_{t+1} - [\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + \ln(b\Pi)]\}B_t + \frac{\omega_{t+1}}{b^{t+1}}$ with probability of $\Pi$ in regime G; $B_{t+1} = \frac{\omega_{t+1}}{b^{t+1}}$ with probability of (1- $\Pi$ ) in regime C;

where $E_t(\omega_{t+1}) = 0.$ (31)

The bubble process of (31) is a general version of the bursting bubbles suggested by Blanchard and Watson (1982) who restricted the value of $\lambda$ in (31) to be zero. It is noted that the expected value of the bubble in regime G, $E_t(B_{t+1}|G) = (b\Pi)^{-1} B_t$, where $(b\Pi)^{-1} > \boldsymbol{b}^{-1}$, and the bubble will collapse to zero expected value as it bursts, $E_t(B_{t+1}|C) = 0$.

In addition, Evans (1991) suggests a periodically collapsing bubble specification:

$B_{t+1} = \exp[\lambda x_{t+1} - (\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + \ln b)]\ B_t + \frac{\omega_{t+1}}{b^{t+1}}$ for $B_t \leqq \kappa$, and

$$= \exp[\lambda x_{t+1} - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2)][\delta_o + \theta_{t+1}\Pi^{-1}b^{-1}(B_t - \delta_o b)] + \frac{\omega_{t+1}}{b^{t+1}}$$

for $B_t > \kappa$. (32)

where both $\kappa$ and $\delta_o > 0$, $\theta_t$ is an exogenous independently and identically distributed Bernoulli process that takes the value of 1 with probability of $\Pi$ in regime G and 0 with a probability of (1- $\Pi$ ) in regime C. Since $(B_{t-1} - \delta_o b)$ is restricted to be positive, $\delta_o$ must be smaller than ( $\kappa\, \boldsymbol{b}^{-1}$). The collapsing bubble is specified to be positive since if a bubble collapses to zero, it cannot re-start (Diba and Grossman, 1988).

For $B_t \leqq \kappa$, it implies that $\Pi = 1$ and $E_t(B_{t+1}) = \frac{B_t}{b}$. For $B_t > \kappa$, $E_t(B_{t+1} | G) = [\delta_o - \delta_o\Pi^{-1} + \Pi^{-1}b^{-1}(B_t)]$ and $E_t(B_{t+1} | C) = \delta_o > 0$. Hence, $E_t(B_{t+1})$ is equal to $\frac{B_t}{b}$ for any value of $B_t$ and the bubble process of Evans (1991) can satisfy the sub-martingale property. It is found that the collapsing bubble is strictly positive and never vanishes. Moreover, the size of bubble collapse or explosion and the probability $\Pi$ are dependent upon the sizes of the bubble compared to the value of $\kappa$. The probability, $\Pi$, can be a variable as a function of the size of bubble (Van Norden, 1996). Also, the bubble bursts partially in contrast to the total bubble collapse of the bursting bubble of (31).

### 2.2.4 General Solution

The general solution to the difference equation (3), denoted by $\pi_t^g$, is equal to the sum of particular and homogenous solutions, i.e., $\pi_t^f + B_t$. The stochastic process of $\pi_t^f$ characterizes the long-run equilibrium path of $\pi_t^g$; on the other hand, the movement of $B_t$ characterizes the deviation of $\pi_t^g$ from $\pi_t^f$. If the model under study is correctly specified, the task of bubble testing consists in detecting whether any movements of asset price deviate from the paths predicted by the market fundamental solution.

## 2.3 CONCLUSION

This paper is slightly modified from Chapters One and Two of Woo (2004), in which two versions of the Cagan model under rational expectations are specified. The general solution of the Cagan model is simply the sum of fundamental and bubble solutions. The fundamental solutions can be expressed in explicit representations dependent upon the assumed generating processes of the underlying fundamentals. There exist arbitrary martingales in the bubble solution, which is therefore non-unique in the rational expectations model. By restricting parameters of the bubble solution, several examples of different theoretical bubble specifications can be explored. Some exhibit asymptotic stability and some display different switching behaviours under alternate regimes of explosion and collapse. It makes the indirect testing methodologies more attractive for bubble detection.

## COMPETING INTERESTS

Author has declared that no competing interests exist.

## REFERENCES

Blanchard, O. J. and Watson, M. W. (1982), “Bubbles, Rational Expectations, and Financial Markets,” in Paul Wachtel (eds.) Crisis in the Economic Financial Structure, MA: Lexington Books.

Broze, L., Gourieroux, C. and Szafarz, A. (1995). Solutions of Multivariate Rational Expectations Models, Econometric Theory, 11, 229-257.

Broze, L. and A. Szafarz, (1991). The Econometric Analysis of Non-Uniqueness in Rational Expectations Models, Amsterdam: North Holland.

Cagan, P. (1956). The Monetary Dynamics of Hyperinflation. In M. Friedman (eds.), Studies in the Quantity Theory of Money. Chicago: University of Chicago Press.

Diba, B. T., and H. I. Grossman (1988b), Rational Inflationary Bubbles, Journal of Monetary Economics, 21, 35-46.

Evans, G. E. (1991). Pitfalls in Testing for Explosive Bubbles in Asset Prices, American Economic Review, 81, 922-930.

Flood, R. P. and Garber, P. M. (1980). Market Fundamentals versus Price-Level Bubbles: The First Tests, Journal of Political Economy, 88, 745-770.

Frenkel, J. A. (1977). The Forward Exchange Rate, Expectations and the Demand for Money: The German Hyperinflation, American Economic Review, 67, 653-670.

Frenkel, J. A. (1979). Further Evidence on Expectations and the Demand for Money During German Hyperinflation, Journal of Monetary Economics, 5, 81-96.

Funke, M., Hall, S. and Sola, M. (1994). Rational Bubbles During Poland’s Hyperinflation: Implications and Empirical Evidence, European Economic Review, 38, 1257-1276.

Garber, P. (1989). Tulipmania, Journal of Political Economy, 97, 535-560.

Garber, P. (1990). Famous First Bubbles, Journal of Economic Perspective, 4, 35-54.

Granger, C. W. J. and Swanson, N. R. (1997). An Introduction to Stochastic Unit-Root Process, Journal of Econometrics, 80, 35-62.

Gourieroux, C., Laffont, J.J. and Monfort, A. (1982). Rational Expectations in Dynamic Linear Models: Analysis of the Solutions, Econometrica, 50, 409-425.

Haldrup, N. (1998). An Econometric Analysis of I(2) Variables, Journal of Economic Surveys, 12, 595-650.

Ikeda, S. and Shibata, A. (1992). Fundamentals-dependent Bubbles in Stock Prices, Journal of Monetary Economics, 30, 143-168.

Ikeda, S. and Shibata, A. (1995). Fundamentals Uncertainty, Bubbles and Exchange Rate Dynamics, Journal of International Economics, 38, 199-222.

Keynes, J. M. (1936). The General Theory of Employment, Interest, and Money, Macmillan.

Kindleberger, C. P. (1987). Bubbles. New Palsgrave Dictionary, Vol. 1, 20-22.

Moosa, I. A. (1999). Testing the Currency-Substitution Model under the German Hyperinflation, Journal of Economics, 70, 61-78.

Obstfeld, M., & Rogoff, K. (1983). Speculative hyperinflations in maximizing models: Can we rule them out? Journal of Political Economy, 91(4), 675-687.

Okina, K. (1984). Rational Expectation, Bubbles and Foreign Exchange Market, Bank of Japan Monetary and Economic Studies, 2, 81-118.

Salge, M. (1997). Rational Bubbles. Berlin: Springer.

Sargent, T. J. (1977). The Demand for Money during Hyperinflations under Rational Expectations: I, International Economic Review, 18, 59-82.

Sargent, T. J. (1982), “The Ends of Four Big Inflations”, in: Robert E. Hall (Ed.), Inflation: Causes and Effects. Chicago University Press.

Sargent, T. J. and Wallace, N. (1973). Rational Expectations and the Dynamics of Hyperinflation, International Economic Review, 14, 328-50.

Taylor, M. P. (1991). The Hyperinflation Model of Money Demand Revisited, Journal of Money, Credit and Banking, 23, 327-351.

Tirole, J. (1982). On the Possibility of Speculation under Rational expectations, Econometrica, 50, 1163-1181.

Tirole, J. (1985). Asset Bubbles and Overlapping Generations. Econometrica, 53, 1499-1528.

Van Norden, S. (1996). Regime Switching as a Test for Exchange Rate Bubbles, Journal of Applied Econometrics, 11(3), 219-251.

Weil, P. (1989). Overlapping Families of Infinitely-lived Agents, Journal of Public Economics, 38, 183-198.

Woo, Kai-Yin (2004), Empirical Testing for Bubbles during the Inter-War European Hyperinflations, PhD Thesis, University of Stirling, Scotland, UK.

# APPENDIX 1

Given that $|b|<1$, the presented discounted value of $X_t$ can be expressed as follows:

$$\sum_{i=0}^{\infty} b^i E_t(X_{t+i}) = X_t + bE_t(X_{t+1}) + b^2E_t(X_{t+2}) + b^3E_t(X_{t+3}) + \dots$$
$$= X_t + bE_t(\Delta X_{t+1}) + b^2E_t(\Delta X_{t+2}) + b^3E_t(\Delta X_{t+3}) + \dots$$
$$+bX_t + b^2E_t(\Delta X_{t+1}) + b^3E_t(\Delta X_{t+2}) + \dots$$
$$= X_t + \sum_{i=1}^{\infty} b^i E_t(\Delta X_{t+i}) + b \sum_{i=0}^{\infty} b^i E_t(X_{t+1+i})$$
$$= \frac{X_t}{1-b} + \frac{1}{1-b}\sum_{i=1}^{\infty} b^i E_t(\Delta X_{t+i}) \qquad \text{(A1.1)}$$

Suppose that $\Delta X_{t+j} = \mu + \omega(t+\mathrm{j}) + \eta_{t+j}$, the values of $b^i E_t(\Delta X_{t+i})$ are given as:

$bE_t(\Delta X_{t+1}) = b\mu + b\omega t + b\omega$ $\qquad$ $b^2E_t(\Delta X_{t+2}) = b^2\mu + b^2\omega t + 2b^2\omega$

$b^3E_t(\Delta X_{t+3}) = b^3\mu + b^3\omega t + 3b^3\omega$ $\qquad$ $b^nE_t(\Delta X_{t+n}) = b^n\mu + b^n\omega t + nb^n\omega$ as $n \to \infty$

Hence, $\sum_{i=1}^{\infty} b^i E_t(\Delta X_{t+i})$ is equal to the sum of the following three components: $\sum_{i=1}^{\infty} b^i\mu$ + $\sum_{i=1}^{\infty} b^i\omega t$ + $\sum_{i=1}^{\infty} ib^i\omega$.

The value of each component is calculated as follows:

$$\sum_{i=1}^{\infty} b^i\mu = \frac{b\mu}{1-b}, \quad \sum_{i=1}^{\infty} b^i\omega t = \frac{b\omega t}{1-b}, \quad \sum_{i=1}^{\infty} ib^i\omega = \frac{1}{(1-b)}\sum_{i=1}^{\infty} b^i\omega = \frac{b\omega}{(1-b)^2} \qquad \text{(A.1.2)}$$

It is known that $\frac{1}{1-b} = 1-\beta$, and $\frac{b}{1-b} = -\beta$, then, from (A1.1) and (A1.2), it is found that:

$$\sum_{i=0}^{\infty} b^i E_t(X_{t+1+i}) = \frac{X_t}{1-b} + \frac{1}{1-b}[\frac{b\mu}{1-b} + \frac{b\omega t}{1-b} + \frac{b\omega}{(1-b)^2}]$$
$$= (1-\beta)X_t + (1-\beta)[-\beta\mu - \beta\omega t - \beta(1-\beta)\omega]$$
$$= (1-\beta)X_t - \beta(1-\beta)[\mu + \omega t + (1-\beta)\omega]$$
$$= (1-\beta)X_t + \beta(\beta-1)\{\mu + \omega[(1-\beta) + t]\}$$

# APPENDIX 2

Given that $\Delta^2 X_{t+j}$ = $\mu + \omega_1(t+\text{j}) + \omega_2(t+\text{j})^2 + \eta_{t+j}$ , the values of $b^i E_t(\Delta X_{t+i})$ are shown as:

$bE_t(\Delta X_{t+1})$ $= b\Delta X_t + b\mu + b\omega_1(t+1) + b\omega_2(t+1)^2$
$b^2E_t(\Delta X_{t+2})$ $= b^2\Delta X_{t+1} + b^2\mu + b^2\omega_1(t+2) + b^2\omega_2(t+2)^2$
$= b^2\Delta X_t + 2b^2\mu + b^2\omega_1(t+1) + b^2\omega_2(t+1)^2 + b^2\omega_1(t+2) + b^2\omega_2(t+2)^2$
$b^3E_t(\Delta X_{t+3})$ $= b^3\Delta X_t + 3b^3\mu + b^3\omega_1(t+1) + b^3\omega_2(t+1)^2 + b^3\omega_1(t+2) +$
$b^3\omega_2(t+2)^2 + b^3\omega_1(t+3) + b^3\omega_2(t+3)^2$
$b^nE_t(\Delta X_{t+n})$ $= b^n\Delta X_t + nb^n\mu + b^n\omega_1(t+1) + b^n\omega_2(t+1)^2 + b^n\omega_1(t+2) + b^n\omega_2(t+2)^2 +$
$b^n\omega_1(t+3) + b^n\omega_2(t+3)^2 + .. + b^n\omega_1(t+n) + b^n\omega_2(t+n)^2$ as $n \to \infty$

From the above, $\sum_{i=1}^{\infty} b^i E_t(\Delta X_{t+i})$ is equal to the sum of the following four components:

$$\sum_{i=1}^{\infty} b^i \Delta X_t + \sum_{i=1}^{\infty} ib^i\mu + \sum_{i=1}^{\infty} b^i\omega_1[\Pi_{j=1}^{i}(t+j)] + \sum_{i=1}^{\infty} b^i\omega_2[\Pi_{j=1}^{i}(t+j)^2]$$

The finite values of the above four components of $\sum_{i=1}^{\infty} b^i E_t(\Delta X_{t+i})$ are derived as follows:

$$\sum_{i=1}^{\infty} b^i \Delta X_t = \frac{\Delta X_t}{1-b},$$

$$\sum_{i=1}^{\infty} ib^i\mu = \frac{1}{1-b}\sum_{i=1}^{\infty} b^i\mu = \frac{b\mu}{(1-b)^2},$$

$$\sum_{i=1}^{\infty} b^i\omega_1[\Pi_{j=1}^{i}(t+j)]$$

$$= \sum_{i=1}^{\infty} t(ib^i\omega_1) + \sum_{i=1}^{\infty} b^i\omega_1(\sum_{j=1}^{i} j)$$

$$= \frac{1}{1-b}\sum_{i=1}^{\infty} t(b^i\omega_1) + (b\omega_1 + (1+2)b^2\omega_1 + (1+2+3)b^3\omega_1 + ...)$$

$$= \frac{1}{(1-b)^2} b\omega_1 t + \frac{1}{1-b}\sum_{i=1}^{\infty} ib^i\omega_1$$

$$= \frac{1}{(1-b)^2} b\omega_1 t + \frac{1}{(1-b)^2}\sum_{i=1}^{\infty} b^i\omega_1$$

$$= \frac{1}{(1-b)^2} b\omega_1 t + \frac{b\omega_1}{(1-b)^3},$$

$$\sum_{i=1}^{\infty} b^i \omega_2 [\Pi_{j=1}^{i} (t+j)^2]$$

$$= \sum_{i=1}^{\infty} t^2 (i b^i \omega_2) + \sum_{i=1}^{\infty} t b^i \omega_2 (\sum_{j=1}^{i} 2j) + \sum_{i=1}^{\infty} b^i \omega_2 (\sum_{j=1}^{i} j^2)$$

$$= \frac{1}{1-b} \sum_{i=1}^{\infty} t^2 (b^i \omega_2) + \frac{1}{1-b} \sum_{i=1}^{\infty} t(2ib^i)\omega_2 + \frac{1}{(1-b)} \sum_{i=1}^{\infty} (i^2) b^i \omega_2$$

$$= \frac{t^2 b \omega_2}{(1-b)^2} + \frac{1}{(1-b)^2} \sum_{i=1}^{\infty} t(2b^i)\omega_2 + \frac{1}{(1-b)^2} \sum_{i=1}^{\infty} (2i-1) b^i \omega_2$$

$$= \frac{t^2 b \omega_2}{(1-b)^2} + \frac{t(2b)\omega_2}{(1-b)^3} + \frac{b\omega_2}{(1-b)^3} \sum_{i=1}^{\infty} 2b^i \omega_2$$

$$= \frac{t^2 b \omega_2}{(1-b)^2} + \frac{t(2b)\omega_2}{(1-b)^3} + \frac{b\omega_2 + b^2 \omega_2}{(1-b)^4}. \quad \text{(A.2.1)}$$

From (A.1.1) and (A.2.1), the value of $\sum_{i=0}^{\infty} b^i E_t(X_{t+i})$ is equal to:

$$\frac{X_t}{1-b} + \frac{1}{1-b}\left[\frac{\Delta X_t}{1-b} + \frac{b\mu}{(1-b)^2} + \frac{b\omega_1 t}{(1-b)^2} + \frac{b\omega_1}{(1-b)^3} + \frac{t^2 b\omega_2}{(1-b)^2} + \frac{t(2b)\omega_2}{(1-b)^3} + \frac{b\omega_2 + b^2\omega_2}{(1-b)^4}\right]$$

$$= \frac{X_t}{1-b} + \frac{\Delta X_t}{(1-b)^2} + \frac{b}{(1-b)^3}\left[\mu + \frac{\omega_1}{(1-b)} + \frac{\omega_2(1+b)}{(1-b)^2}\right] + \frac{b}{(1-b)^3}\left[\omega_1 + \frac{2\omega_2}{(1-b)}\right] t + \frac{b\omega_2}{(1-b)^3} t^2$$

$$= (1-\boldsymbol{\beta})\boldsymbol{X_t} + (1-\boldsymbol{\beta})^2 \Delta \boldsymbol{X_t} - \boldsymbol{\beta}(1-\boldsymbol{\beta})^2 \{\mu + (1-\boldsymbol{\beta})\omega_1 + \omega_2[(1-\boldsymbol{\beta})^2 - \boldsymbol{\beta}(1-\beta) + \boldsymbol{t}^2] + \boldsymbol{t}[\omega_1 + 2\omega_2(1-\boldsymbol{\beta})]\}$$

# APPENDIX 3

Let $x_t = w_t - w_{t-1}$. Substituting one period forward for $B_{t+1}$ from the general bubble specification (15), it is found that:

$$B_{t+1} = \exp[\lambda x_{t+1} - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)]\ B_t + \frac{\omega_{t+1}}{b^{t+1}}$$

$$= \exp[\lambda w_{t+1} - \lambda w_t - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)]\ B_t + \frac{\omega_{t+1}}{b^{t+1}}$$

$$= \exp[\lambda w_{t+1} - \lambda w_t - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)(t+1) + (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)(t+1)$$
$$-(\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + ln\, b)]\ B_t + \frac{\omega_{t+1}}{b^{t+1}}$$

$$= \exp\{\lambda w_{t+1} - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)(t+1) - [\lambda w_t - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)]t\}B_t + \frac{\omega_{t+1}}{b^{t+1}}$$

Assume that $\{\omega_t\} = 0$, then:

$$\frac{B_{t+1}}{B_t} = \frac{\exp[\lambda w_{t+1} - \lambda(\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)(t+1)]}{\exp[\lambda w_t - \lambda(\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)(t)]},$$

Hence, $B_t = \exp[\lambda w_t - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2 + \ln b)t]$. (A.3.1)

By imposing different parameter restrictions upon (A.3.1), I can obtain different bubble specifications summarized in Table 2.1.

## APPENDIX 4

Let's examine whether the linear combination of bubble process (19a) and (19b) or the bubble process (22a) can still satisfy the sub-martingale property such that: $bE_{t-1}(B_t) = E_{t-1}(q_t B_{t-1})$.

It is known that $bE_{t-1}(B_t) = E_{t-1}[\exp(\ln b)B_t]$. By substituting the bubble process (22a) into $E_{t-1}[\exp(\ln b)B_t]$, I obtain:

$$\begin{aligned} bE_{t-1}(B_t) &= E_{t-1}\{\exp(\ln b)[\mathrm{A}_1 \exp(\lambda_1 w_t - \mathrm{H}t) + \mathrm{A}_2 \exp(\lambda_2 w_t - \mathrm{H}t)]\} \\ &= E_{t-1}\{\mathrm{A}_1 \exp[\lambda_1 w_t - \mathrm{H}t + \ln b] + \mathrm{A}_2 \exp[\lambda_2 w_t - \mathrm{H}t + \ln b]\} \end{aligned}$$

Also, it is known that $E_{t-1}(q_t B_{t-1}) = E_{t-1}(\exp[\lambda x_t - (\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2)]B_{t-1})$. By substituting the bubble process (22a) into $E_{t-1}(\exp[\lambda x_t - (\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2)]B_{t-1})$, I therefore obtain:

$$\begin{aligned} E_{t-1}(q_t B_{t-1}) = {} & E_{t-1}\{\exp[\lambda_1 w_t - \lambda_1 w_{t-1} - (\lambda\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2)]\mathrm{A}_1 \exp[\lambda_1 w_{t-1} - \mathrm{H}(t-1)] + \\ & \exp[\lambda_2 w_t - \lambda_2 w_{t-1} - (\lambda_2\mu_x + \tfrac{1}{2}\lambda^2\sigma_x^2)]\mathrm{A}_2 \exp[\lambda_2 w_{t-1} - \mathrm{H}(t-1)]\} \end{aligned}$$

Given the assumption that $\ln b$ = K + H, and $(\lambda\mu_x + \frac{1}{2}\lambda^2\sigma_x^2 + K)$ = 0, then:

$$\begin{aligned} E_{t-1}(q_t B_{t-1}) &= E_{t-1}\{\mathrm{A}_1 \exp[\lambda_1 w_t - (\lambda_1\mu_x + \tfrac{1}{2}\lambda_1^2\sigma_x^2 + \mathrm{K}) - \mathrm{H}t + \ln b] + \\ &\qquad \mathrm{A}_2 \exp[\lambda_2 w_t - (\lambda_2\mu_x + \tfrac{1}{2}\lambda_2^2\sigma_x^2 + \mathrm{K}) - \mathrm{H}t + \ln b]\} \\ &= E_{t-1}\{\mathrm{A}_1 \exp[\lambda_1 w_t - \mathrm{H}t + \ln b] + \mathrm{A}_2 \exp[\lambda_2 w_t - \mathrm{H}t + \ln b]\} = bE_{t-1}(B_t) \end{aligned}$$

# APPENDIX 5

Given the fact that $\exp(\pm i \mathrm{h}_2 w_t)$ = $[\cos(\mathrm{h}_2 w_t) \pm i \sin(\mathrm{h}_2 w_t)]$, after substituting $\lambda_1$ = $h_1$ + $h_2$ ***i***, and $\lambda_2$ = $h_1$ - $h_2$ ***i*** into (22a), I obtain:

$B_t$ = $\exp(\text{-H}t)[\mathrm{A}_1 \exp(\mathrm{h}_1 + \mathrm{h}_2 i) w_t + \mathrm{A}_2 \exp(\mathrm{h}_1 - \mathrm{h}_2 i) w_t]$

= $\exp(\mathrm{h}_1 w_t \text{-H}t)[\mathrm{A}_1 \exp(i \mathrm{h}_2 w_t) + \mathrm{A}_2 \exp(-i \mathrm{h}_2 w_t)]$

= $\exp(\mathrm{h}_1 w_t \text{-H}t)\{\mathrm{A}_1[\cos(\mathrm{h}_2 w_t) + i \sin(\mathrm{h}_2 w_t)] + \mathrm{A}_2[\cos(\mathrm{h}_2 w_t) - i \sin(\mathrm{h}_2 w_t)]\}$

= $\exp(\mathrm{h}_1 w_t \text{-H}t)[(\mathrm{A}_1 + \mathrm{A}_2)\cos(\mathrm{h}_2 w_t) + (\mathrm{A}_1 - \mathrm{A}_2) i \sin(\mathrm{h}_2 w_t)]$

= $\exp(\mathrm{h}_1 w_t \text{-H}t)[\mathrm{A}_3 \cos(\mathrm{h}_2 w_t) + \mathrm{A}_4 \sin(\mathrm{h}_2 w_t)]$.

where $A_3=A_1+A_2$, $A_4 = (A_1-A_2)$***i***

**Biography of author(s)**

**Kai-Yin Woo (Associate Professor)**
Department of Economics and Finance, Hong Kong Shue Yan University, Hong Kong.

He is an Associate Professor at the Department of Economics and Finance of Hong Kong Shue Yan University. He obtained PhD at the University of Stirling, Scotland. His research interests include the economics of finance and applied econometrics. His research topics include bubble testing, investigation of price convergence and purchasing power parity, and empirical study on the Belt and Road Initiative and the Greater Bay Area of China. He has published journal articles in Economics Letters, Applied Economics, Economic Modelling, Journal of Macroeconomics, Singapore Economic Review, International Review of Economics and Finance, Advances in Decision Sciences, Applied Economics Letters, Empirical Economics and others.

---



**Peer-Review History:**
This chapter was reviewed by following the Advanced Open Peer Review policy. This chapter was thoroughly checked to prevent plagiarism. As per editorial policy, a minimum of two peer-reviewers reviewed the manuscript. After review and revision of the manuscript, the Book Editor approved the manuscript for final publication. Peer review comments, comments of the editor(s), etc. are available here: https://peerreviewarchive.com/review-history/12411F-CH2

***Chapter 3***
Print ISBN: 978-81-973195-8-7, eBook ISBN: 978-81-973195-0-1

# Markov-Switching Cointegration Test for Bubbles during the Interwar European Hyperinflations

**Kai-Yin Woo** [a++*]

DOI: 10.9734/bpi/mono/978-81-973195-8-7/CH3

**Peer-Review History:**
This chapter was reviewed by following the Advanced Open Peer Review policy. This chapter was thoroughly checked to prevent plagiarism. As per editorial policy, a minimum of two peer-reviewers reviewed the manuscript. After review and revision of the manuscript, the Book Editor approved the manuscript for final publication. Peer review comments, comments of the editor(s), etc. are available here: https://peerreviewarchive.com/review-history/12411F-CH3

## ABSTRACT

The purpose of this paper is to test for the presence of price and exchange rate bubbles in Cagan's model using data from the interwar European hyperinflations of Germany, Hungary, and Poland. Markov-switching cointegration test would be adopted for the empirical analysis. Then, the regime-shifting behaviour of time series variables is assumed to depend on unobservable states generated by a first-order Markov chain. The probability law that governs the Markov-switching regimes is advantageous in that it is more flexible and allows the data to determine the specific form of nonlinearities that are consistent with the sample information. Inferences about the probabilities of the unobservable states at each point in time can also be made.

*Keywords: Cagan model; price bubbles; exchange rate bubbles; Markov-switching.*

**JEL classifications:** C5, F4.

## 3.1 INTRODUCTION

The existence of speculative bubbles has been an issue of long-standing debate. Many studies have applied Cagan's (1956) model to test for evidence of price bubbles during hyperinflation. This problem has probably attracted so much attention with far-reaching policy implications. Particularly, the choice of an appropriate policy to deal with hyperinflation very much depends on the true nature of the underlying inflation. If rational bubbles are not present, then it is

---

[a] *Department of Economics and Finance, Hong Kong Shue Yan University, Hong Kong.*
[++] *Associate Professor;*
**Corresponding author: E-mail: kywoo@hksyu.edu;*

only necessary to take control of the market fundamentals. If, however, inflation is being driven by a bubble phenomenon, then positive action is needed to shock expectations from the bubble path. Moreover, it is noted that the expectations and implementation of monetary reforms during periods of hyperinflation might lead to regime changes in economic variables. Failure to model the regime-shifting behaviour of time series may lead to biased conclusions with respect to cointegration and the existence of bubbles. For example, threshold cointegration analysis was employed to model the switching processes that are however restricted to depend upon observable threshold values (Chan and Woo, 2006). However, threshold nonlinearity and Markovian regime shifts may be observationally equivalent.

The empirical study of bubbles is suggested to be undertaken in hyperinflationary episodes (Flood and Garber, 1980) and the classic examples include the inter-war European hyperinflations of Germany, Hungary and Poland (Sargent, 1982). The purpose of this paper is to test for the presence of bubbles in Cagan's model using data from the interwar European hyperinflations of Germany, Hungary, and Poland while both price and exchange rate bubbles would be tested (Chan et al. 2003; Chan and Woo, 2006; Hooker, 2000). Markov-switching cointegration test would be adopted for the empirical analysis. Then, the regime-shifting behaviour of time series variables is assumed to depend on unobservable states generated by a first-order Markov chain. The probability law that governs the Markov-switching regimes is advantageous in that it is more flexible and allows the data to determine the specific form of nonlinearities that are consistent with the sample information. Inferences about the probabilities of the unobservable states at each point in time can also be made.

The paper is structured as follows: Section 2 contains a discussion of the econometric methodology; Section 3 describes the data; Section 4 reports the empirical results; concluding remarks are contained in Section 5.

## 3.2 ECONOMETRIC METHODOLOGY

Krolzig (1996, 1997), and Yao and Attali (2000) argue that a linear cointegration method is asymptotically valid to test for the number of cointegrating vectors in a Markov error-correction model. However, Nelson *et al.* (2001), Psaradakis (2001) and Cavaliere (2003) consider that the conventional unit root tests will result in biased conclusions when the series under study exhibit Markov shifts. Cavaliere (2003) suggests that unit root or cointegration tests be carried out using a statistical method that allows for the Markov-switching process.

In this paper, the cointegration-testing procedure of Engsted (1993) will be conducted for the identification of bubbles by sequentially applying the Markvo-switching-augmented Dickey-Fuller (MS-ADF) test of Hall et al. (1999) to the OLS residuals of the Cagan models. The MS-ADF cointegration methodology can be used to simultaneously test for the existence of nonstationary roots and allow for the possibility of Markovian regime shifts in the structure of the disequilibrium errors. The Markov shifts in the series under study can be detected by allowing the ADF parameters to switch values between different regimes generated by a Markov process. Further, the simulation study conducted by Hall et al. (1999) confirms that, compared to the standard ADF test, the MS-ADF t-test statistics can effectively detect the periodically collapsing bubbles of Evans (1991) by identifying the existence of an explosive root at least in one regime. Using the MS-ADF unit root test for empirical studies, Funke et al. (1994) and Hall et al. (1999) found some evidence of inflationary bubbles in the data for Poland from 1991-1993 and for Argentina from 1983 to 1989 respectively.

The stochastic version of the Cagan models under rational expectations is specified as follows.

$$M_t - \pi_{1,t} = \alpha_1 + \beta_1 E_t(\Delta\pi_{1t+1}) + u_{1,t} \quad (1)$$

$$M_t - \pi_{2,t} = \alpha_2 + \beta_2 E_t(\Delta\pi_{2,t+1}) + u_{2,t} \quad (2)$$

where $M_t$ is the natural logarithm of the money stock at time t; $\pi_{1,t}$ and $\pi_{2,t}$ refer to the natural logarithm of the price level and exchange rate, respectively; $E_t(.)$ denotes the mathematical expectations operator conditional on information set; $u_{1,t}$ and $u_{2,t}$ refer to a money demand disturbance.

By defining the rational expectation forecasting errors as $\eta_{j,t+1} = \Delta\pi_{j,t+1} - E_t(\Delta\pi_{j,t+1})$, which are assumed to be a white noise and then serially uncorrelated, Equations.(1) and (2) can be re-written as:

$$M_t - \pi_{j,t} = \alpha_j + \beta_j \Delta\pi_{j,t+1} + \xi_{j,t};\ j = 1, 2, \quad (3)$$

where $\xi_{j,t} = u_{j,t} - \beta_j \eta_{j,t+1}$

For notional convenience, the subscript j is eliminated in subsequent equations. Re-arranging Equations (1) and (2) in terms of $\pi_t$ , recursively substituting forward for $E_t(\pi_{t+1+i})$ using the law of iterated expectations, and imposing the non-bubble transversality condition, Equation (3) can be re-written as:

$$(M_t - \pi_t) = \alpha + \beta\Delta M_t + (\beta-1)\sum_{i=1}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(\Delta^2 M_{t+i}) + (\frac{1}{1-\beta})\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(u_{t+i}). \quad (4)$$

Engsted (1993) proposes a set of sequential testing procedures to detect the presence of $B_t$. The procedure is conducted by comparing the cointegrating relationship between $M_t - \pi_t$ and $\Delta\pi_{t+1}$ in Equation (3) as well as between $M_t - \pi_t$ and $\Delta M_t$ in Equation (4). The Cagan models given by Equation (3) represent a money market equilibrium condition that admits a general solution. Therefore, provided that $u_t$ is stationary, the Cagan models cannot be rejected even if $B_t$ exists. On the other hand, the non-bubble transversality condition is imposed upon Equation (4) but not upon Equation (3). Hence, Equation (4) represents a fundamental solution only, which can be rejected if $B_t$ is present in the price or exchange rate data.

Let's start with the general form of the MS-ADF regression of order p with 2 regimes, which allows for different regime shifts in the parameters, although the inclusion of regime-dependent deterministic trend may capture the explosive dynamics of a bubble process:

$$\Delta y_t = c(s_t) + \rho(s_t) y_{t-1} + \sum_{j=1}^{p-1} \Psi_j(s_t)\Delta y_{t-i} + b(s_t)t + e_t$$
$$e_t \mid s_t \sim \text{NID}(0, \sigma_e(s_t)), \qquad t = 1 \ldots T. \qquad (5)$$

where $y_t$ is an OLS residual of Equation (3) or (4), $e_t$ is a white noise process with zero mean and regime-switching standard deviation $\sigma_e(s_t)$. All parameters of the autoregression, $c(s_t)$, $\rho(s_t)$, $\psi_j(s_t)$, and $\sigma_e(s_t)$ are conditioned on a finite number of stochastic unobservable Markov-switching state variable $s_t \in \{1,2\}$ such that:

$$c(s_t) = c_1 s_{1t} + c_2 s_{2t}, \quad \rho(s_t) = \rho_1 s_{1t} + \rho_2 s_{2t}, \quad b(s_t) = b_1 s_{1t} + b_2 s_{2t},$$
$$\psi_j(s_t) = \psi_{j1} s_{1t} + \psi_{j2} s_{2t}, \quad \sigma_e(s_t) = \sigma_1 s_{1t} + \sigma_2 s_{2t} \qquad (6)$$

where $s_{it}$ takes on the value 1 when $s_t$ = ***i***, and 0 otherwise, for ***i*** = 1, 2.

The stochastic process generating the unobservable regimes is an ergodic Markov chain governed by the transition probabilities, $\boldsymbol{P_{ij}}$ = Pr [$s_t = j \mid s_{t-1} = i$]

with $\sum_{j=1}^{2} P_{ij}$ = 1 $\forall$ i, j $\in$ {1,2}. For an ergodic Markov chain, regime shifts are persistent if $P_{ij} \neq P_{ii}$ for some i ≠j, but not permanent if $P_{ii} \neq 1$ $\forall$ i. The filtered probability of $s_t = j,$ denoted by $\Pr[s_t = j \mid \Omega_t],$ is equal to $\sum_{i=1}^{2} \Pr[s_t = j, s_{t-1} = i \mid \Omega_t]$, conditional on information up to time t, $\Omega_t$. The smoothed probability of $s_t = j,$ denoted by $\Pr[s_t = j \mid \Omega_T],$ conditional on all the information in the sample at time T, $\Omega_T,$ and is calculated by $\sum_{i=1}^{2} \Pr[s_t = j, s_{t+1} = i \mid \Omega_T]$. The Hamilton's (1989) filtering and the Kim's (1994) smoothing algorithms are employed to make inferences about the filtered and smoothed probabilities of the unobservable Markov regimes respectively. An expectation-maximization (EM) algorithm for maximum likelihood estimation is used to yield estimated parameters of the MS-ADF model (5).

The existence of nonstationary roots is rejected when the two individual t-ratio statistics, $t_1$ and $t_2$, reject the null hypothesis that $\rho_i \geqq 0$ against the alternative of $\rho_i < 0$, for all i = 1 and 2. Yao and Attali (2000) document more rigorous stability conditions for a Markvo-switching model. In addition to the t-ratio statistics, a Wald statistic is proposed to test for the joint hypothesis that $\rho_1$ = $\rho_2$ = 0 against the alternative of $\rho_i \neq 0$ for at least one i. When both the point estimates of $\rho_1$ and $\rho_2$ lie in the open interval of -2 and 0, the significance of the Wald statistic implies the rejection of the existence of nonstationary roots across two regimes in the data series of interest. The associated p-values of the Wald and t-ratio statistics are obtained via simulation.

On the other hand, the number of the Markov-switching regimes cannot be tested using conventional testing approaches due to the presence of unidentified nuisance parameters such as the transition probabilities under the null of linearity. Hansen (1992, 1996) derived formal tests of Markov-switching nonlinearity, which involve the approximation of the asymptotic distribution of the likelihood ratio (LR) via simulation and evaluation of the likelihood function across a grid of different values for the transition probabilities as well as for each state-dependent parameter. This is, however, computationally demanding and time-consuming. In practice, Krolzig (2002) suggests alternatives that include the upper bound of Davies (1977, 1987) for the significance level of the LR test statistics under nuisance parameters, and information criteria such as AIC, SC and HQ (see for example, Krolzig, *et al.* 2002 and Clarida, *et al.* 2003). Actually, the Wald statistic for the joint hypothesis that the intercept terms are equal across two regimes is valid (Krolzig, *et.al.* 2002). Hence, if a singular matrix occurs when the LR test is implemented, the Wald statistic will be used instead.

## 3.3 DATA

The data include data series of money supply, price index levels, and exchange rate series for the inter-war European hyperinflations of Germany, Hungary and Poland. The German exchange rate series and all the data for Hungary and Poland are taken from Young (1925), while the German money supply and price index are collected from Tinbergen (1934). The money supply series are month-end data, whereas the other series are monthly averages; I, therefore, follow Abel et al. (1979) in applying the geometric averaging method to make the money supply series conform to the rest of the data. Also, all of the exchange rate series that are originally quoted as the number of US cents per unit of local currency are transformed in terms of the values of domestic currency per US dollar. The German data used for estimation cover the periods from January 1920 up to June 1923; for Hungary, the data sets include the periods from July 1921 to February 1924 and the Polish data cover the period between January 1921 and December 1923. All data series in the samples are transformed in a natural logarithm.

Using structural time series modelling techniques (Harvey, 1989), money supply $M_t$, price indexes $\pi_{1,t}$ and exchange rate series $\pi_{2,t}$ are found to contain double unit roots. It is consistent with Haldrup (1998) that the economic variables are likely to be I(2) during hyperinflation. Moreover, the real money balances $M_t - \pi_t$, the first-differenced money supply $\Delta M_t$, price indexes $\Delta\pi_{1,t}$ and exchange rate series $\Delta\pi_{2,t}$ have a stochastic trend, implying that these series contain a unit root. Hence, all variables in Equations (3) and (4) are I(1). Then, we can test for the cointegration in Equations (3) and (4), and examine if the residuals are I(0). The results of the structural time series analysis of data are all available in Chapter Three of Woo (2004).

## 3.4 EMPIRICAL RESULTS

In the subsequent empirical studies, the estimation is conducted using the maximum likelihood method with the expectation-maximization algorithm. The intercept terms $c(s_t)$, standard deviation of error $\sigma_e(s_t)$, and the coefficients $b(s_t)$, $\psi_j(s_t)$, and $\rho(s_t)$, of the MS-ADF regression (5), can be made regime-dependent if the corresponding upper bound LR test statistic of Davies (1977,1987) is significant. The number of lag length, p, is chosen to make $e_t$ white noise. The p-values of the MS-ADF Wald and t-ratio statistics are obtained via simulation. For some cases, a vector of regime-dependent stationary covariates is added to the MS-ADF regressions to increase statistical power.

**Table 3.1(a). Cointegrating MS-ADF test for the regression of $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$**

| **Country** | **Germany** | | **Hungary** | | **Poland** | |
|---|---|---|---|---|---|---|
| **Parameters** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** |
| **Intercept of the MS-ADF regression:** | | | | | | |
| $c(s_t)$ | 0.2028*** (0.1072) | | 1.0282* (0.2199) | 0.0918 (0.0769) | 1.0687* (0.0859) | 0.1634 (0.1576) |
| LR $(c_1 = c_2)$ | 0.0025# [0.9599] | | 18.4070* [0.0004] | | 8.7839** [0.0323] | |
| **Coefficients of the ADF regression:** | | | | | | |
| $b(s_t)$ | -0.0046 (0.0037) | -0.0294* (0.0053) | -0.0508* (0.0078) | -0.0006 (0.0021) | -0.0495* (0.0026) | -0.0021 (0.0069) |
| $\rho(s_t)$ | -0.2697 | -1.5341 | -0.8473 | -0.2527 | -0.7355 | -0.6644 |
| **t** $(\rho(s_t))$ | -4.3517* | -6.7925* | -6.4622* | -3.8816** | -25.1221* | -3.8598** |
| W$(\rho(s_t))$ | 66.0191* | | 51.4814* | | 652.829* | |
| Joint LR | 51.0888* [0.0000] | | 25.6972* [0.0001] | | 12.1681** [0.0300] | |
| **Standard error of residuals** | | | | | | |
| $\sigma_e(s_t)$ | 0.0695 | 0.9889 | 0.1200 | 0.1153 | 0.0370 | 0.2731 |
| LR $(\sigma_1 = \sigma_2)$ | 53.6782* [0.0000] | | 14.4253* [0.0024] | | 9.1883** [0.0269] | |
| **Transition probability matrix:** | | | | | | |
| P11 P12 | 0.8310 0.1690 | | 0.4837 0.5173 | | 0.4110 0.5890 | |

| **Country** | **Germany** | | **Hungary** | | **Poland** | |
|---|---|---|---|---|---|---|
| **Parameters** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** |
| P21 P22 | 0.6773 0.3227 | | 0.1729 0.8271 | | 0.1816 0.8184 | |
| | | | **Diagnostic checking** | | | |
| AIC | -0.0332 | | -0.3292 | | 0.6719 | |
| HQ | 0.1663 | | -0.1782 | | 0.8250 | |
| SC | 0.5386 | | 0.0804 | | 1.1208 | |
| Q(12-p) | 8.1515 | | 7.9668 | | 9.8592 | |

*Notes:*

***t*** *( $\rho(s_t)$ ) and W( $\rho(s_t)$ ) refer to the MS-ADF t-ratio and Wald statistics.*

*Joint LR refers to the joint LR linearity test for the coefficients $\psi_1(s_t)$, $b(s_t)$ and $\rho(s_t)$, of the MS-ADF regression.*

*# Denotes the Wald statistic, rather than the LR statistic, for linearity tests.*

*The figures in (.) are standard errors. The figures in [.] are the p-values for the significance of* ***t*** *( $\rho(s_t)$ ), W( $\rho(s_t)$ ) and the upper bound LR linearity tests.*

*AIC, HQ and SC refer to the Akaike, Hannan and Quinn, and Schwarz criterion, respectively.*

*Q(k) refers to Ljung-Box Q-statistics with degrees of freedom = k.*

*For the case of Germany, the stationary covariates include $\Delta^2\pi_{2,t-1}$, and $\Delta^2 M_{t-1}$.*

**/**/*** Denote significance at the 1%, 5% and 10% level.*

**Table 3.1b. Cointegrating MS-ADF test for the regression of $(M_t - \pi_{1,t})$ on $\Delta M_t$**

| **Country** | **Germany** | | **Hungary** | | **Poland** | |
|---|---|---|---|---|---|---|
| **Parameters** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** |
| | | | **Intercept of the ADF regression:** | | | |
| $c(s_t)$ | 0.6770* | | -0.1264** | 0.1889* | 0.1935* | |
| | (0.1069) | | (0.0499) | (0.0477) | (0.0164) | |
| LR $(c_1 = c_2)$ | 0.0000# | | 11.2275** | | 0.0003# | |
| | [0.9932] | | [0.0106] | | [0.9855] | |
| | | | **Coefficients of the ADF regression:** | | | |
| $\psi_1(s_t)$ | ________ | | 0.5581* | | -0.0385 | 0.6612* |
| | | | (0.0873) | | (0.0444) | (0.1968) |
| $\psi_2(s_t)$ | ________ | | ________ | | 0.3992* | 0.6127* |
| | | | | | (0.0452) | (0.2025) |
| $b(s_t)$ | -0.0258* | -0.0249* | -0.0027** | | -0.0178* | -0.0044* |
| | (0.0036) | (0.0042) | (0.0013) | | (0.0008) | (0.0014) |
| $\rho(s_t)$ | -0.3968 | -0.4378 | -0.6259 | | -0.2293 | -0.8422 |
| **t** ( $\rho(s_t)$ ) | -5.0306* | -4.4110* | -8.5393* | | -7.9263* | -5.6670* |
| W( $\rho(s_t)$ ) | 33.5772* | | 72.9204* | | 87.431* | |
| Joint LR | 50.4305* | | 9.4629 | | 35.2855* | |

| **Country** | **Germany** | | **Hungary** | | **Poland** | |
|---|---|---|---|---|---|---|
| **Parameters** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** |
| | [0.0000] | | [0.2284] | | [0.0000] | |
| **Standard error of residuals** | | | | | | |
| $\sigma_e(s_t)$ | 0.0562 | 0.1384 | 0.09442 | | 0.0175 | 0.1200 |
| LR $(\sigma_1 = \sigma_2)$ | 32.5865*<br>[0.0000] | | 0.0232<br>[0.8789] | | 19.4283*<br>[0.0002] | |
| **Transition probability matrix** | | | | | | |
| P11 P12 | 0.5937 0.4063 | | 0.8625 0.1375 | | 0.4373 0.5627 | |
| P21 P22 | 0.5332 0.4668 | | 0.0769 0.9231 | | 0.2889 0.7111 | |
| **Diagnostic checking** | | | | | | |
| AIC | -0.8563 | | -0.9086 | | -0.7081 | |
| HQ | -0.6568 | | -0.7873 | | -0.4804 | |
| SC | -0.2786 | | -0.5777 | | -0.0211 | |
| Q(12-p) | 13.2538 | | 12.7802 | | 10.8851 | |

*Notes:*

*For the cases of Germany and Poland, the stationary covariates include* $\Delta^2 M_{t-i}$ *, for i = 1, 2, and* $\Delta^2 \pi_{2,t-1}$ *respectively.*

*Other notes to Table 3.1(a) still apply.*

### 3.4.1 Price Bubbles

Table 3.1(a) contains the maximum likelihood estimates of the MS-ADF regressions for the OLS residuals of Equation (3), which are obtained from regressing $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$. As pointed out by Yao and Attali (2000), the OLS estimates of a cointegration vector remain super-consistent under regular conditions even though the disequilibrium error exhibits a Markov-switching process. The estimation results show that all parameters of the MS-ADF regression are made regime-dependent for all of the hyperinflations under study, except the fact that the intercept term is regime-invariant for Germany. All the point estimates of $\rho_1$ and $\rho_2$ in two regimes are negative and the corresponding Wald and t-ratio statistics are significant from the associated p-values. Hence, the results favor the evidence of cointegrating relationships between $M_t - \pi_{1,t}$ and $\Delta\pi_{1,t+1}$. Table 3.1(b) presents the estimation results of the MS-ADF test for the OLS residuals obtained from regressing $(M_t - \pi_{1,t})$ on $\Delta M_t$. As shown in Table 3.1(b), the intercepts of the MS-ADF regression are allowed to be regime-dependent for Hungary only. The coefficients $b(s_t)$, $\psi_j(s_t)$, and $\rho(s_t)$ can vary across different regimes for Germany and Poland. The standard deviation of $e_t$ can be regime-dependent for Germany and Hungary. All point estimates of $\rho_1$ and $\rho_2$ are negative, and both the Wald and t-ratio statistics are significant. This signifies the acceptance of the cointegrated relationships of $(M_t - \pi_{1,t})$ on $\Delta M_t$. When $(M_t - \pi_{1,t})$ cointegrates with $\Delta\pi_{1,t+1}$ and $\Delta M_t$ the evidence for price bubbles in the data, as suggested by Engsted (1993), is rejected.

Figs. 3.1 to 3.6 plot the filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$ , and $(M_t - \pi_{1,t})$ on $\Delta M_t$, from which the Markovian regime shifts are found in the OLS residuals of Equations (3) and (4) throughout the whole samples.

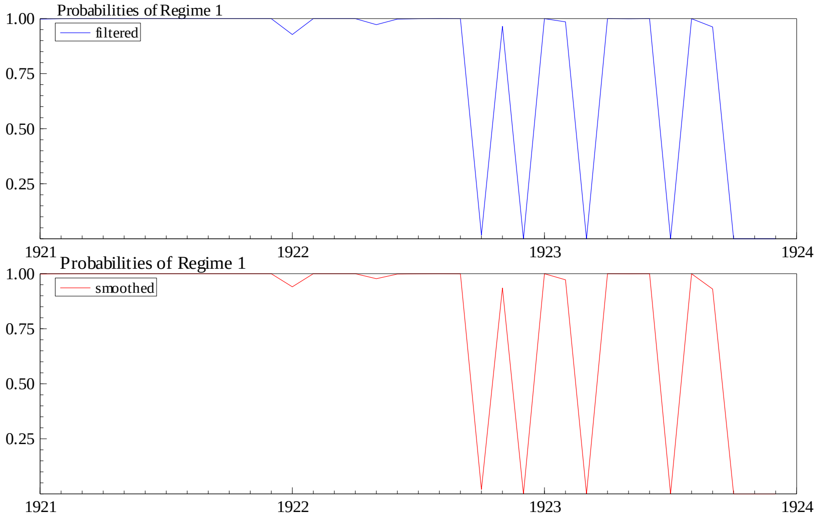



**Fig. 3.1. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$ in Germany**

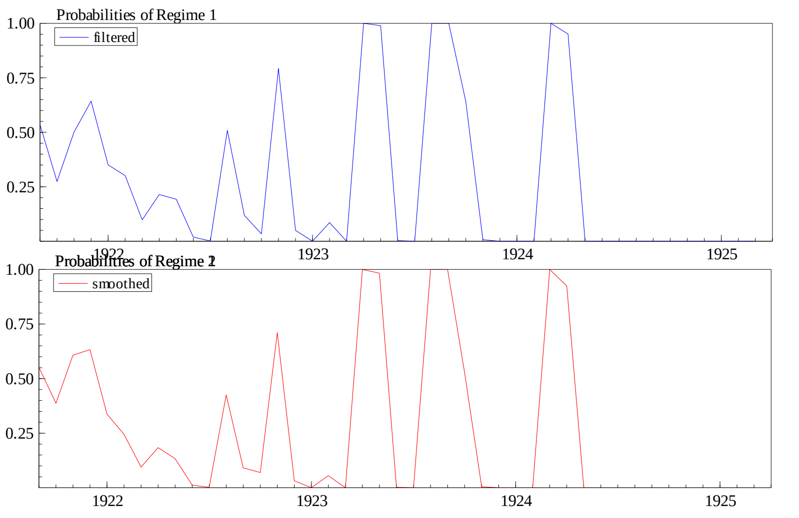



**Fig. 3.2. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$ in Hungary**

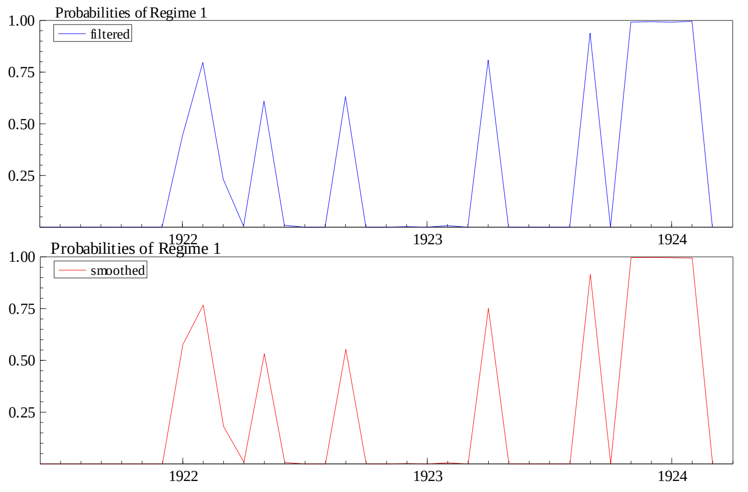



**Fig. 3.3. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on $\Delta\pi_{1,t+1}$ in Poland**

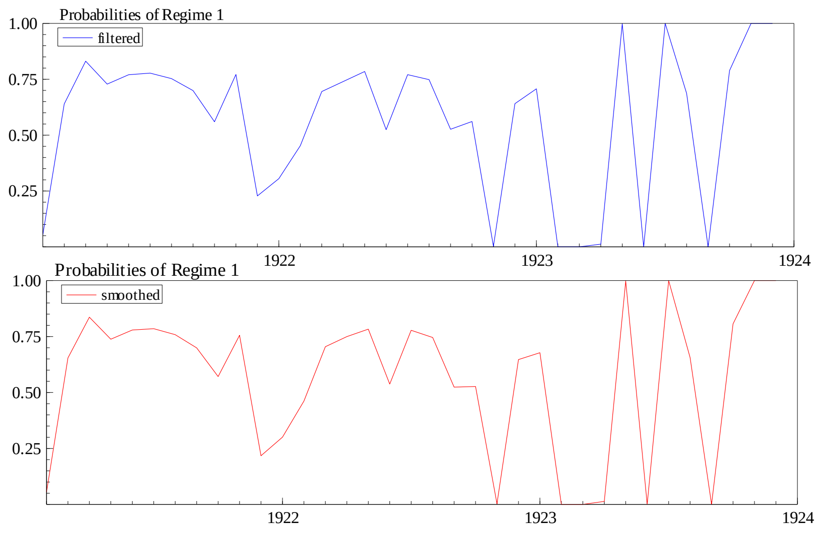



**Fig. 3.4. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on $\Delta M_t$ in Germany**

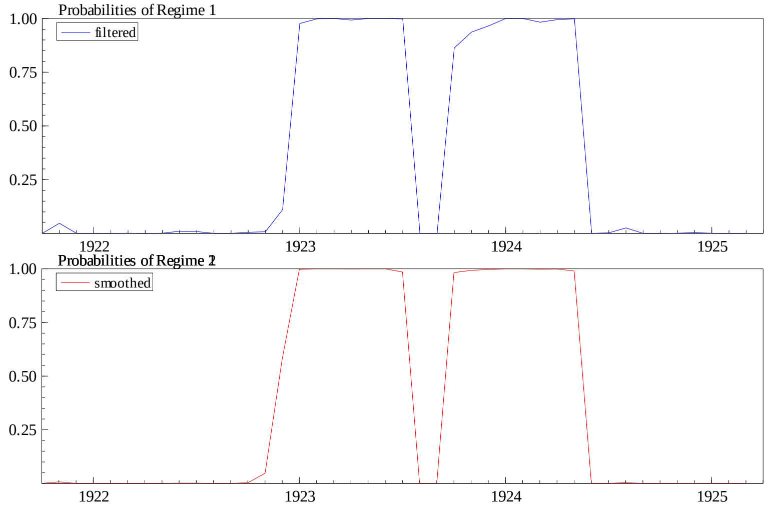



**Fig. 3.5. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on $\Delta M_t$ in Hungary**

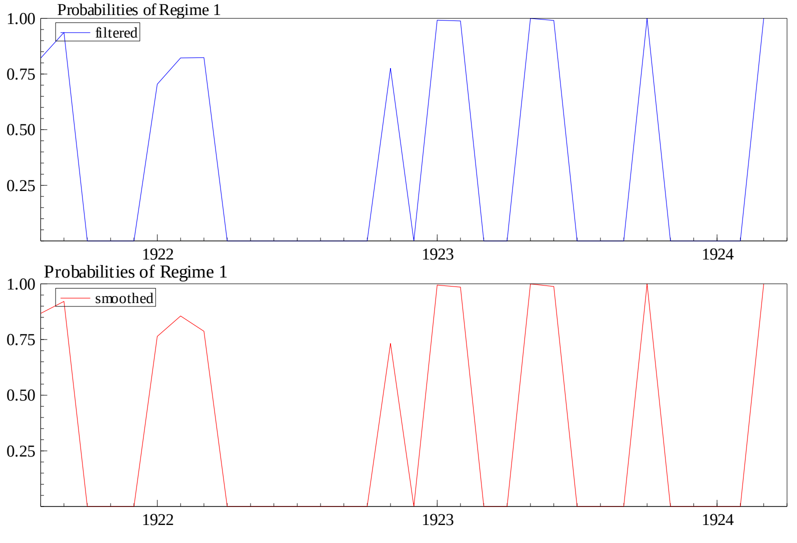



**Fig. 3.6. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{1,t})$ on $\Delta M_t$ in Poland**

**Table 3.2(a). Cointegrating MS-ADF test for the regression of $(M_t - \pi_{2,t})$ on $\Delta\pi_{2,t+1}$**

| **Country** | **Germany** | | **Hungary** | | **Poland** | |
|---|---|---|---|---|---|---|
| **Parameters** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** |
| | | | **Intercept of the ADF regression:** | | | |
| $c(s_t)$ | 2.1098** | 1.3878* | -0.8864* | -0.0460 | 0.4547* | |
| | (0.7046) | (0.3523) | (0.0676) | (0.0791) | (0.1595) | |
| LR $(c_1 = c_2)$ | 9.2272** | | 65.0991*# | | 3.6115 | |
| | [0.0264] | | [0.0000] | | [0.3066] | |
| | | | **Coefficients of the ADF regression:** | | | |
| $\psi_1(s_t)$ | ——— | | -0.7453* | 0.1874** | ——— | |
| | | | (0.0956) | (0.0738) | | |
| $\psi_2(s_t)$ | ——— | | 0.2020** | 0.2024* | ——— | |
| | | | (0.0936) | (0.0627) | | |
| $b(s_t)$ | -0.0909* | -0.0412* | 0.0237* | 0.0044*** | -0.0217* | |
| | (0.0191) | (0.0114) | (0.0018) | (0.0023) | (0.0070) | |
| $\rho(s_t)$ | -1.0375 | -0.6391 | -0.1768 | -0.2710 | -0.7360 | |
| **t** ( $\rho(s_t)$ ) | -6.8660* | -5.1620* | -4.0354** | -3.8991** | -4.5379* | |
| W( $\rho(s_t)$ ) | 73.8228* | | 32.1445* | | 20.6209* | |
| Joint LR | 33.0602* | | 44.9593* | | 6.6673 | |
| | [0.0000] | | [0.0000] | | [0.5732] | |

| **Country** | **Germany** | | **Hungary** | | **Poland** | |
|---|---|---|---|---|---|---|
| **Parameters** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** |
| | | | **Standard error of residuals** | | | |
| $\sigma_e(s_t)$ | 0.4678 | 0.1119 | 0.0344 | 0.0849 | 0.3107 | |
| LR $(\sigma_1 = \sigma_2)$ | 22.7773*<br>[0.0000] | | 6.7481***<br>[0.0804] | | 2.3391<br>[0.5051] | |
| | | | **Transition probability matrix** | | | |
| P11 P12 | 0.7322 0.2678 | | 0.1909 0.8091 | | ______ | |
| P21 P22 | 0.1516 0.8484 | | 0.3819 0.6181 | | | |
| | | | **Diagnostic checking** | | | |
| AIC | 0.8359 | | -0.6609 | | 0.7222 | |
| HQ | 1.0201 | | -0.3847 | | 0.7836 | |
| SC | 1.3637 | | 0.1390 | | 0.8981 | |
| Q(12-p) | 9.0479 | | 11.7997 | | 7.3214 | |

*Notes:*

*For the cases of Germany and Hungary, the stationary covariates are* $\Delta^2\pi_{1,t-1}$, *and* $\Delta^2\pi_{1,t-i}$, *i = 1,2, respectively.*

*Other notes to Table 3.1(a) still apply*

**Table 3.2(b). Cointegrating MS-ADF test for the regression of $(M_t - \pi_{2,t})$ on $\Delta M_t$**

| **Country** | **Germany** | | **Hungary** | | **Poland** | |
|---|---|---|---|---|---|---|
| **Parameters** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** |
| | | | **Intercept of the ADF regression:** | | | |
| $c(s_t)$ | 0.8312*<br>(0.1875) | | 0.1508*<br>(0.0453) | -0.4197*<br>(0.0879) | -0.3880*<br>(0.0667) | 0.0220<br>(0.0548) |
| LR $(c_1 = c_2)$ | 0.0022#<br>[0.9625] | | 32.1756*#<br>[0.0000] | | 9.1718**<br>[0.0271] | |
| | | | **Coefficients of the ADF regression:** | | | |
| $\psi_1(s_t)$ | 1.7130*<br>(0.2334) | -0.4274*<br>(0.0861) | -0.5732*<br>(0.0925) | 0.3166*<br>(0.1085) | ________ | |
| $b(s_t)$ | -0.0236*<br>(0.0070) | -0.0310*<br>(0.0059) | -0.0103*<br>(0.0018) | 0.0141*<br>(0.0026) | 0.0029<br>(0.0023) | |
| $\rho(s_t)$ | -0.5922 | -0.3631 | -0.2790 | -0.4094 | -0.7084 | |
| **t**( $\rho(s_t)$ ) | -4.0692* | -5.0590* | -3.5969** | -4.9534* | -8.1681* | |
| W( $\rho(s_t)$ ) | 34.0088* | | 38.2115* | | 66.7179* | |
| Joint LR | 34.8726*<br>[0.0000] | | 23.1208*<br>[0.0068] | | 10.5068<br>[0.3214] | |
| | | | **Standard error of residuals** | | | |
| $\sigma_e(s_t)$ | 0.2600 | 0.2276 | 0.0463 | 0.0824 | 0.1170 | |
| LR $(\sigma_1 = \sigma_2)$ | 25.9889*<br>[0.0000] | | 12.0458*<br>[0.0072] | | 0.0000#<br>[0.9973] | |

| **Country** | **Germany** | | **Hungary** | | **Poland** | |
|---|---|---|---|---|---|---|
| **Parameters** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** | **Regime 1** | **Regime 2** |
| | | | **Transition probability matrix** | | | |
| P11 P12 | 0.2778 0.7222 | | 0.6893 0.3107 | | 0.5628 0.4372 | |
| P21 P22 | 0.3094 0.6906 | | 0.1540 0.8460 | | 0.1811 0.8189 | |
| | | | **Diagnostic checking** | | | |
| AIC | 1.1407 | | -1.0614 | | 0.0253 | |
| HQ | 1.3095 | | -0.8179 | | 0.1631 | |
| SC | 1.6295 | | -0.3927 | | 0.4293 | |
| Q(12-p) | 11.5172 | | 10.3732 | | 11.1613 | |

*Notes:*

*For the cases of Hungary and Poland, the stationary covariates are equally* $\Delta^2\pi_{1,t-i}$*, i = 1, 2.*

*Other notes to Table 3.1(a) still apply.*

### 3.4.2 Exchange Rate Bubbles

The maximum likelihood estimates of the MS-ADF regressions for the OLS residuals of Equation (3) are obtained from regressing $(M_t - \pi_{2,t})$ on $\Delta\pi_{2,t+1}$. The results are presented in Table 3.2(a). The intercept term of the MS-ADF regression is made state-dependent for Germany and Hungary. The joint LR allows the coefficients $b(s_t)$, $\psi_j(s_t)$, and $\rho(s_t)$, of the MS-ADF regression to switch between regimes for Germany and Hungary. Also, the standard error of $e_t$ can be state-dependent for Germany and Hungary. In other words, all the parameters of the MS-ADF regression for Poland are restricted to be regime-independent and the t-ratio value of $\rho$ is the standard ADF **t** test statistic. For Germany and Hungary, the point estimates of $\rho_1$ and $\rho_2$ in two regimes are found to be negative and the MS-ADF Wald and t-ratio statistics are significant. From the above, it can be concluded that there exists a cointegrating relationship between $(M_t - \pi_{2,t})$ and $\Delta\pi_{2,t+1}$ for all of the countries under study. Moreover, the results of the cointegrating tests applied to the OLS residuals of Eq.(5.6) obtained from regressing $(M_t - \pi_{2,t})$ on $\Delta M_t$ are presented in Table 3.2b. For Germany, only the intercept of the MS-ADF regression is restricted to be state-invariant, whereas for Poland, only the intercept can be made regime-dependent. All the parameters of the MS-ADF regression are different across regimes for Hungary. From the negative point estimates of $\rho_1$ and $\rho_2$ with significant Wald and t-ratio statistics, the existence of a cointegrated relationship between $(M_t - \pi_{2,t})$ and $\Delta M_t$ cannot be rejected. While $(M_t - \pi_{2,t})$ cointegrates with $\Delta\pi_{2,t+1}$ and with $\Delta M_t$, no nonstationary roots are found in the residuals of Equations (3) and (4) respectively. Hence, the evidence for an exchange rate bubble is rejected in the data for all of the hyperinflations under study.

The patterns of regime shifts can be seen in Figs. 3.7 to 3.11 where the filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on $\Delta\pi_{2,t+1}$ and on $\Delta M_t$, are plotted. The regime-switching behaviours can be found throughout the whole estimation period for the countries under study.

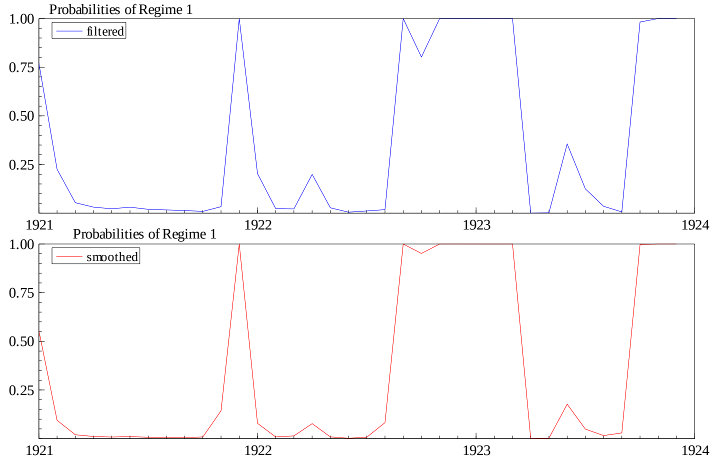



**Fig. 3.7. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on $\Delta\pi_{2,t+1}$ in Germany**

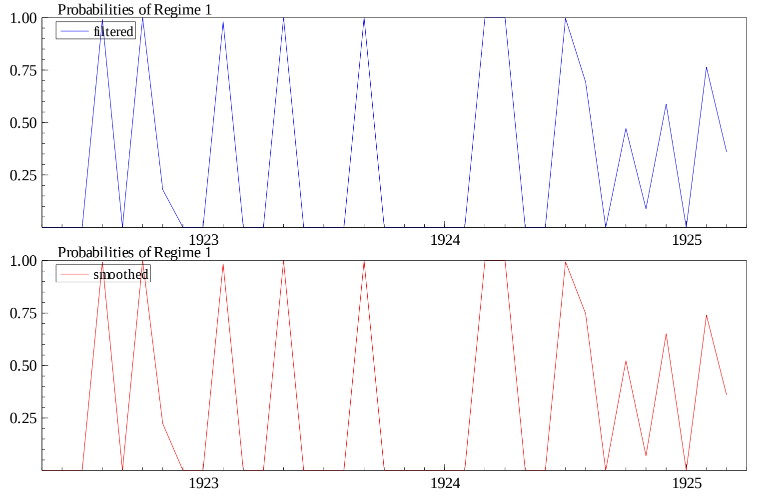



**Fig. 3.8. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on $\Delta\pi_{2,t+1}$ in Hungary**

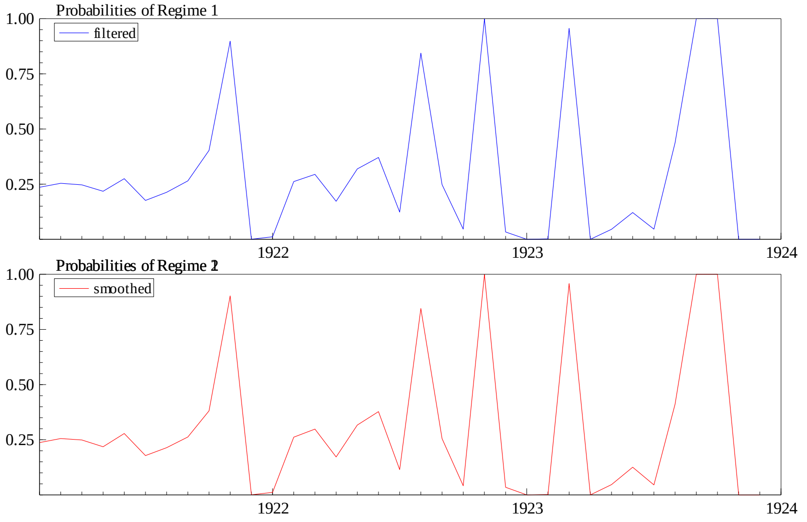



**Fig. 3.9. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on $\Delta M_t$ in Germany**

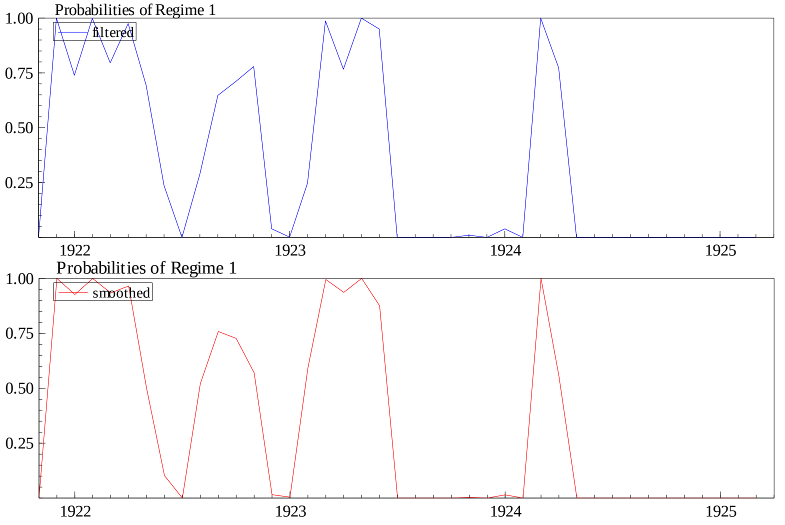



**Fig. 3.10. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on $\Delta M_t$ in Hungary**

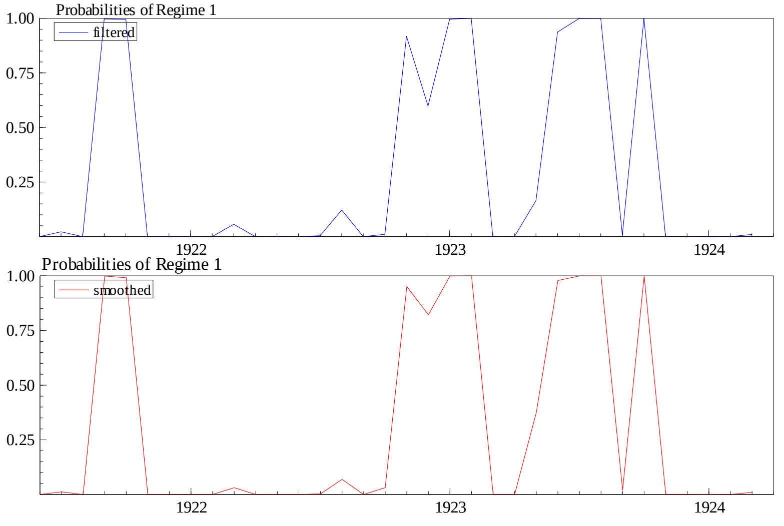



**Fig. 3.11. Filtered and smoothed probabilities of regime 1 for the residuals of the regression of $(M_t - \pi_{2,t})$ on $\Delta M_t$ in Poland**

## 3.5 CONCLUSION

This paper is slightly modified from Chapter Six of Woo (2004), in which the cointegration tests to examine the bubble existence are conducted using the cointegrating MS-ADF tests. The regime shifts in the MS-ADF regression are allowed to depend on an unobservable state variable governed by the Markov chain rather than on an observable threshold value. The empirical results show that the evidence of Markovian regime shifts is found in the cointegration residuals from the regression of $M_t - \pi_t$ and $\Delta\pi_{t+1}$ as well as $M_t - \pi_t$, and $\Delta M_t$. Also, the point estimates of $\rho_i$ are all negative and the Wald and t-ratio statistics are all significant. Hence, the evidence favors the MS cointegrating relationship in both Equations (3) and (4), and it rejects the presence of bubbles in any of the countries under study. On the other hand, more formal tests of bubbles can be attempted such as Phillips et al. (2015ab) in future research.

## COMPETING INTERESTS

Author has declared that no competing interests exist.

## REFERENCES

Abel, A., Dornbusch, R., Huizinga, J. & Marcus, A. (1979). Money Demand During Hyperinflation. Journal of Monetary Economics, 5, 97-104.

Cagan, P. (1956). The Monetary Dynamics of Hyperinflation. In M. Friedman (eds.), Studies in the Quantity Theory of Money. Chicago: University of Chicago Press.

Cavaliere, G. (2003). Asymptotics for Unit Root Tests under Markov Regime-switching. Econometric Journal, 6, 193-216.

Chan, H. L., Lee, S. K., & Woo, K. Y. (2003). An empirical investigation of price and exchange rate bubbles during the interwar European hyperinflations. International Review of Economics & Finance, 12(3), 327-344.

Chan, H. L., & Woo, K. Y. (2006). Bubbles Detection for Inter-war European hyperinflation: A Threshold Cointegration Approach. Journal of Economics and Finance, 30(2), 169-185.

Clarida, R. H., Sarno, L., Talyor, M. P. and Valente, G. (2003). The Out-of-Sample Success of Term structure Models as Exchange Rate Predicators: A Step Beyond. Journal of International Economics, 60, 61-83.

Davies, R. B. (1977). Hypothesis Testing When a Nuisance Parameter is Present only Under the Alternative. Biometrika, 64, 247-254.

Davies, R. B. (1987). Hypothesis Testing When a Nuisance Parameter is Present only Under the Alternative. Biometrika, 74, 33-43.

Engsted, T. (1993). Cointegration and Cagan's model of Hyperinflation under Rational Expectations. Journal of Money, Credit and Banking, 25, 350-360.

Evans, G. E. (1991). Pitfalls in Testing for Explosive Bubbles in Asset Prices. American Economic Review, 81, 922-930.

Flood, R. P. and Garber, P. M. (1980). Market Fundamentals versus Price-Level Bubbles: The First Tests. Journal of Political Economy, 88, 745-770.

Funke, M., Hall, S. and Sola, M. (1994). Rational Bubbles During Poland's Hyperinflation: Implications and Empirical Evidence. European Economic Review, 38, 1257-1276.

Hall, S. G., Psaradakis, Z. and Sola, M. (1999). Detecting Periodically Collapsing Bubbles: A Markov-Switching Unit Root Test. Journal of Applied Econometrics, 14, 143-154.

Haldrup, N. (1998). An Econometric Analysis of I(2) Variables. Journal of Economic Surveys, 12, 595-650.

Hamilton, J. D. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, Econometrica, 57, 357-384.

Hansen, B.E. (1992). The Likelihood Ratio Test under Nonstandard Conditions: Testing the Markov Switching Model of GNP. Journal of Applied Econometrics 7(5), S61-82.

Hansen, B.E. (1996). Erratum: The Likelihood Ratio Test under Nonstandard Conditions: Testing the Markov Switching Model of GNP. Journal of Applied Econometrics, 11(2), 195-98.

Harvey, A.C. (1989). Forecasting, Structural Time Series Models and Kalman Filter. Cambridge University Press

Hooker, M. A. (2000). Misspecification versus Bubbles in Hyperinflation Data: Monte Carlo and Interwar European Evidence. Journal of International Money and Finance, 19, 583-600.

Kim, C. J. (1994). Dynamic Linear Models with Marko-Switching, Journal of Econometrics 60, 1-22.

Krolzig, H. M. (1996). Statistical Analysis of Cointegrated VAR Processes with Markovian Regime Shifts. SFB 373 Discussion Paper, 25/1996, Humboldt Universitat zu Berlin.

Krolzig, H. M. (1997). Markov Switching Vector Autoregression, Modelling, Statistical Inference and Application to Business Cycle Analysis. Berlin: Springer.

Krolzig, H. M. (2002). Regime-Switching Models. Mimo, University of Oxford.

Krolzig, H. M., Marcellino, M. Mizon, G. E. (2002). A Markov-switching Vector Equilibrium Correction Model of the UK Labour Market. Empirical Economics, 27, 233-254.

Nelson, C., Piger, J. and Zivot, E. (2001), Markov Regime Switching and Unit-Root Tests. Journal of Business and Economic Statistics, 19, 404-415.

Phillips, P. C. B., Shi, S., & Yu, J. (2015a). Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500. International Economic Review, 56(4), 1034–1078.

Phillips, P. C. B., Shi, S., & Yu, J. (2015b). Testing for Multiple Bubbles: Limit Theory for Real-Time Detectors. International Economic Review, 56(4), 1079–1134.

Psaradakis, Z. (2001). Markov Level Shift and the Unit Root Hypothesis. Econometrics Journal, 4, 226-42.

Sargent, T. J. (1982), “The Ends of Four Big Inflations”, in: Robert E. Hall (Ed.), Inflation: Causes and Effects. Chicago University Press.

Tinbergen, J. (ed.) (1934). International Abstract of Economic Statistics 1910-30. London: International Conference of Economic Services.

Woo, Kai-Yin (2004), Empirical Testing for Bubbles during the Inter-War European Hyperinflations, PhD Thesis, University of Stirling, Scotland, UK.

Yao, J. F. and Attali, J. G. (2000). On Stability of Nonlinear Processes with Markov Switching. Advances in Applied Probability, 32, 394-407.

Young, J.P. (1925). European Currency and Finance, Vol. 2. Washington, D.C: U.S. Government Printing Office.

**Biography of author(s)**

**Kai-Yin Woo (Associate Professor)**
Department of Economics and Finance, Hong Kong Shue Yan University, Hong Kong.

He is an Associate Professor at the Department of Economics and Finance of Hong Kong Shue Yan University. He obtained PhD at the University of Stirling, Scotland. His research interests include the economics of finance and applied econometrics. His research topics include bubble testing, investigation of price convergence and purchasing power parity, and empirical study on the Belt and Road Initiative and the Greater Bay Area of China. He has published journal articles in Economics Letters, Applied Economics, Economic Modelling, Journal of Macroeconomics, Singapore Economic Review, International Review of Economics and Finance, Advances in Decision Sciences, Empirical Economics, and others.



**Peer-Review History:**
This chapter was reviewed by following the Advanced Open Peer Review policy. This chapter was thoroughly checked to prevent plagiarism. As per editorial policy, a minimum of two peer-reviewers reviewed the manuscript. After review and revision of the manuscript, the Book Editor approved the manuscript for final publication. Peer review comments, comments of the editor(s), etc. are available here: https://peerreviewarchive.com/review-history/12411F-CH3

**_Chapter 4_**
Print ISBN: 978-81-973195-8-7, eBook ISBN: 978-81-973195-0-1

# Investment Behaviour for Small Investors in the Hong Kong Stock Market

**Tai-Yuen Hon [a++*], Wing-Kwong Au [b#]**
**and Wing-Keung Wong [c,d,e†]**

DOI: 10.9734/bpi/mono/978-81-973195-8-7/CH4

**Peer-Review History:**
This chapter was reviewed by following the Advanced Open Peer Review policy. This chapter was thoroughly checked to prevent plagiarism. As per editorial policy, a minimum of two peer-reviewers reviewed the manuscript. After review and revision of the manuscript, the Book Editor approved the manuscript for final publication. Peer review comments, comments of the editor(s), etc. are available here: https://peerreviewarchive.com/review-history/12411F-CH4

## ABSTRACT

We extend Hon's (2012) paper to identify and analyse the important factors that capture the behaviour of small investors in the Hong Kong stock market, especially during the financial crisis. Exploratory factor analysis is employed to analyze the data, we find that the reference group is the most important factor and monitoring investments is the second important factor.

*Keywords: Stock market; marketing analysis; investment behaviour.*

## 4.1 INTRODUCTION

Hong Kong is a small open economy. It is common to find that some small investors have done less rational things in the financial markets, especially when investing in stocks. The primary objective is to identify and analyse the important factors that capture the behaviour of small investors in the Hong Kong stock market. It is important to find out whether their investment behaviour can be explained by some underlying factors grounded in the behavioural approach to the study of financial markets. In our study, secondary data were not available to facilitate our research. Our study data were collected primarily through a survey questionnaire directed (face-to-face) at small investors. A group of undergraduate students helped to

---

[a] *Business, Economic and Public Policy Research Centre, Hong Kong Shue Yan University, Hong Kong.*
[b] *Department of Social Work, Hong Kong Shue Yan University, Hong Kong.*
[c] *Department of Finance, Fintech & Blockchain Research Center, and Big Data Research Center, Asia University, Taiwan.*
[d] *Department of Medical Research, China Medical University Hospital Business, Taiwan.*
[e] *Economic and Public Policy Research Centre, Hong Kong Shue Yan University, Hong Kong.*
[++] *Research Affiliate;*
[#] *Associate Professor;*
[†] *Chair Professor;*
**Corresponding author: E-mail: tyhon@hksyu.edu;*

distribute 1,200 questionnaires to the respondents. There were 1,199 selected respondents who were successfully interviewed. Hon (2012) concluded that small investors were overconfident and bought more stock during the buoyant market in the Hong Kong stock market. Small investors also exhibited herd behaviour. Exploratory factor analysis was employed to analyse the data. In doing so, we hope to extend Hon's paper and contribute to the study of behavioural finance in the context of an Asian financial centre, namely Hong Kong. This chapter's tables and figure are referred to Hon (2015) for Tables 4.1-4.7 and Fig. 4.1.

## 4.2 LITERATURE REVIEW

Li and Ahn (2024) explore the impact of individual investor sentiment derived from social networks on stock market returns. Using keyword-based techniques, they collect and analyze Sina Weibo posts related to COVID-19, extracting daily influential weighted sentiment indexes from a dataset of over 2.4 million posts in 2020. Empirical tests utilizing a sentiment-augmented three-factor model reveal that individual investor sentiment exerts an independent influence on Chinese financial markets, after controlling for market risk, size, and value effects. They further find that negative sentiment carries a stronger impact on stock returns, which is in line with the loss-averse behavior commonly observed among individual investors. They also find an asymmetric pattern in the sentiment-return relation across different industry types. While positive sentiment affects both types of industries that suffer or benefit from COVID-19, negative sentiment affects only the industries that suffer from the pandemic. Overall, the empirical results provide robust support for the significance of individual investor sentiment in explaining the behavior of the Chinese financial markets. This paper introduces a measure of regret for stock market investors (REG) and examines its cross-sectional asset pricing implications. Following an extension of the modified expected utility function from Bell (1982) and Loomes and Sugden (1982), they propose a novel regret measure for stock investments and show that the comparison of a stock's realized return with the best-foregone return that could have been obtained by investing in a similar stock is an important factor in determining investors' modified utility as it captures the variation in investors' current wealth with the foregone wealth opportunity. Using this key variable, they investigate whether REG predicts the cross-sectional variation in future stock returns. Varshini and Vinayalaxmi (2024) look at how loss aversion, overconfidence, professional experience, and investment volume affect investor behavior. This study concludes that the dynamics of investor behavior and its relationship to a variety of psychological, experiential, and market-related variables. Understanding these processes enables investors to make better judgments, financial analysts to construct more accurate models, and regulators to create more effective regulations. Xia and Madni (2024) find five key behavioral factors, including herding, market, prospect, overconfidence, and anchoring-ability bias, influence the investing choices of investors in Chinese stock markets. Four behavioral aspects of the herding factor are all connected to imitating the activities of other investors. Price variations, market knowledge, and previous stock movement are the three elements that make up the market factor. The prospect factor consists of four components: mental accounting (with two sub-variables), mental accounting (with loss aversion and regret aversion), and mental accounting. Two components make up the heuristic variables: overconfidence-gambler's fallacy and anchoring-ability bias. In contrast to the anchoring-ability bias component, which consists of two variables: ability bias and anchoring, the overconfidence-gambler's fallacy factor consists of two variables: overconfidence and gambler's fallacy. The decisions that investors make are affected by all these variables taken together. Arisoy, Bali, and Tang (2024) first document that regret is positively related to the cross-section of future equity returns. Sorting individual stocks into value-weighted portfolios based on their REG shows that stocks with high regret outperform stocks with low regret. The results indicate that regret-averse investors dislike (prefer) stocks that generate high (low) regret because investing in such stocks decreases investors' utility more (less) than other stocks. As a result, stocks with high (low) regret earn higher (lower) future returns in equilibrium. Second, the positive relation between regret and expected returns is robust to using alternative factor models in the calculation of risk-adjusted returns (alphas), different portfolio weighting schemes, controlling for a number of stock characteristics, screening out small, illiquid, and low-priced stocks, and over different periods and stock samples. Multivariate Fama and MacBeth (1973) regressions that simultaneously control for individual stock characteristics further corroborate our main finding that regret is an important determinant of the cross-sectional dispersion in equity returns. Third, using household trading data and following the intuition behind our proposed regret framework for stock investments, they develop an investor-based regret index (REGINDEX) and show that REGINDEX predicts stock returns in a similar way to their proposed regret measure. Fourth, they construct alternative regret measures using longer estimation windows ranging from 2 to 12 months and document that investor regret premium is robust across different estimation periods. Fifth, they document that regret is a highly persistent phenomenon and has a cross-sectional predictive ability that goes beyond one month extending up to five months. Regret is also a distinct investor characteristic that is not spanned by established risk or behavioral factor models. Finally, they investigate the economic underpinnings of the observed regret premium and find that investors do regret by holding unattractive stocks or missing the opportunity to hold attractive stocks due to costly arbitrage and informational frictions.

## 4.3 METHODOLOGY

Factor analysis is employed to identify the key factors that affect the behaviour of small investors in the Hong Kong stock market. In the factor analysis, a standard score on a data item can be expressed as a weighted sum of the common factor scores, the specific factors scores, and the error factor scores. That is,

$$z_{ik} = a_{i1}F_{1k} + a_{i2}F_{2k} + \cdots + a_{im}F_{mk} + a_{is}S_{ik} + a_{ie}E_{ik} \quad (1)$$

where

$z_{ik}$ is a standard score for small investor k on data item i,

$a_{i1}$ is a factor loading for data item i on common factor 1,

$a_{i2}$ is a factor loading for data item i on common factor 2,

$a_{im}$ is a factor loading for data item i on the last common factor,

$a_{is}$ is a factor loading for data item i on specific factor i,

$a_{ie}$ is a factor loading for data item i on error factor i,

$F_{1k}$ is a standard score for small investor k on common factor 1,

$F_{2k}$ is a standard score for small investor k on common factor 2,

$F_{mk}$ is a standard score for small investor k on common factor m, the last common factor,

$S_{ik}$ is a standard score for small investor k on specific factor i,

$E_{ik}$ is a standard score for small person k on error factor i.

Equation (1) may be represented in schematic matrix form for all values of i and k simultaneously, that is, for all data items and all small investors or other data-producing objects. The schematic matrix equation could be represented by the following matrix equation:

$$\mathbf{Z} = A_u F_u \tag{2}$$

Equation (2) states that the matrix of data-item scores **Z** may be obtained by multiplying the matrix of factor loading $A_u$ by the matrix of factor scores $F_u$. The common factor portion of $A_u$ will be called matrix **A** (without the subscript u), and the common factor portion of $F_u$ will be called matrix **F**. We make the factor structure more interpretable. The initial extracted factor matrix must be rotated before the final factor solution is achieved. A factor matrix may be transformed to a rotated factor matrix by matrix operation **V** = **A**Λ, where **V** is rotated matrix, **A** is the unrotated matrix, and Λ is an orthogonal transformation matrix in which rows and columns have sums of squares equal to 1.0 and inner products of non-identical rows or columns equal to zero. Such a transformation does not affect the capacity of the factor matrix to reproduce the original correlation matrix because

$$\mathbf{VV'} = (\mathbf{A}\Lambda)\,(\mathbf{A}\Lambda)' = \mathbf{A}\Lambda\Lambda'\mathbf{A'} = \mathbf{AIA'} = \mathbf{A\,A'} = \mathbf{R} \tag{3}$$

In other words, the transformed or rotated matrix **V** when multiplied by its transpose **V’** will reproduce the **R** matrix just as well as **A** multiplied by its transpose **A’** does. These rotations are carried out using “positive manifold” and “simple structure,” rotational criteria that have been traditional guides in carrying out the rotation process in factor analysis. Trying to rotate to obtain nonnegative loadings is known as rotating to “positive manifold”. The idea behind positive manifold is that if the entire data item in a matrix have inter-correlations that are either zero or positive, it is unreasonable to anticipate an underlying factor with substantial negative loadings for any of data items. Thurstone (1947) developed the criterion of “simple structure” to guide the investigator in carrying out rotations of factor axes to positions of greater “psychological meaningfulness”. Bartlett’s test of sphericity and Kaiser-Meyer-Olkin measure of sampling adequacy are both tests that can be used to determine the factorability of the matrix as a whole. If Bartlett’s test of sphericity is large and significant and the Kaiser-Meyer-Olkin measure is greater than 0.6, then factorability is assumed. If the sums of squares of the loadings on the extracted factors are no longer dropping but are remaining at a low and rather uniform level, factor extraction may be reasonably terminated. Cattell’s (1966) Scree test is based on this principle. SPSS use a default option of extracting all principal factors with eigenvalues of 1.0 or more (i.e., the Kaiser-Guttman rule). The main thing to consider in deciding when to stop factoring is that it is better to err on the side of extracting too many factors rather than too few. One of the most commonly used is Cronbach’s coefficient α, which is based on the average correlation of items within a reliability test if the items are standardised. Cronbach’s coefficient α can be interpreted as a correlation coefficient; it ranges in value from 0 to 1.

## 4.4 DATA

The data for the present study were collected from small investors in Hong Kong through a survey questionnaire. The main purpose of the survey is to collect their opinions, investment behaviour, and financial decision-making behaviour in the stock market. The survey was conducted between October and November 2008. Since the majority of Hong Kong’s population is Chinese, the questionnaire was written in Chinese. After a pilot test on ten respondents, some amendments were made before we finalised the questionnaire. The snowball method was adopted to select individuals aged 18 or above in the Hong Kong population. This sampling technique is often used in hidden populations which are difficult for us to access; snowball sampling uses a small pool of initial informants to nominate, through our university networks, other participants who meet the eligibility criteria and could potentially contribute to this study. The term “snowball sampling” reflects an analogy to a snowball increasing in size as it rolls downhill. (Morgan, 2008). A group of undergraduate students helped to distribute 1,200 questionnaires to the respondents. The target population is the small investors in the Hong Kong stock market. Finally, there were 1,199 selected respondents who completed and returned the questionnaires and this represents a response rate of 99.92 per cent. Since some respondents did not reply to all the questions in the questionnaire, we only used the number of replies (i.e., the questions that respondents did not answer were not counted) to calculate the total number of and the percentage of the total for the individual entries. The questionnaire was designed to elicit information about demographics, investment experience and behaviour, and factors affecting the financial decision-making of the respondents. We took an existing questionnaire developed by Johnsson, Lindblom and Platan (2002) in Lund University, Sweden, and modified it for this study. The first part of the questionnaire focused on the respondents’ investment experience and perceptions about the investment conditions, and the factors that affect their financial decision-making. The second part collected the respondents’ personal information, including gender, age, employment status, and average monthly income.

## 4.5 RESULTS

The profile of the respondents is reported in Table 4.1. The majority of the respondents were under the age of 50 (85.6%), and only 14.4% were aged 51 or above. The median income was $11,660. In view of the above demographic profile of the respondents, we believe that they are representative of small investors in Hong Kong. The importance of various items on the behaviour of small investors when they invest in the stock market is presented in Table 4.2. All the items are statistically significant with high mean values. To identify the underlying dimensions of the items, which are perceived to be important by the respondents, the 10 items were then factor analysed. An initial visual assessment of the correlation matrix indicated a considerable degree of inter-factor correlation (see Table 4.3). In addition, from the correlation matrix, the Barlett test of Sphericity ($\rho < 0.000$) and the Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy index (with a value of 0.546) confirm the appropriateness of the data for exploratory factor analysis (EFA). Given that our aim was to identify the minimum number of factors that would account for the maximum portion of the variance of original items, the principal component analysis was selected (Nunnally, 1978) to reduce the number of factors with an eigenvalue greater than 1. A cumulative percentage of variance explained as being greater than 50% is the criteria used in determining the number of factors. On the basis of the criteria, five factors were extracted (see Table 4.4). The five factors, collectively, accounted for a satisfactory 67.547% of the variance. Communality values between 1.0 and 0 indicate partial overlapping between the items and the factors in what they measure. Furthermore, the commonality column, provides further evidence of the overall significance, albeit, moderate, of the solution. The underlying rationale for the Scree test is based on the fact that within a set of items, a limited number of factors are measured more precisely than the others. By graphing the eigenvalues, we found that the smaller factors form a straight line sloping downward. The dominant factors will fall above the line. Fig. 4.1 demonstrates a five-factor solution is obtained. Having established that the analysis has provided a stable solution, an examination of the varimax-rotated factor loading was performed (see Table 4.5). The cumulative factors revealed that the first factor accounts for 18.768% of the variance. The second factor accounts for 34.219% of the variance. The third factor accounts for 46.897% of the variance. The fourth factor accounts for 57.417% of the variance. Finally, the fifth factor accounts for 67.547% of the variance. After the rotation, there are no negative loadings on any consequence of Factor I, Factor II, Factor III, Factor IV or Factor V. We found five factors affecting the behaviour of small investors in the Hong Kong stock market as follows: factor A might be interpreted as reference group which includes commentators' recommendations from newspapers/TV/magazines, relatives/friends, Internet, investment consultants, and companies' annual reports; factor B as monitor investments which includes monitor short-term and long-term investments; factor C as a personal background which includes age, personal income; factor D as a reaction to announcements which includes announcements and other information from companies, forecasting the future market development and factor E as a cognitive style which includes a factor for a bear market and reason for investment failure.The specific name given to each factor is designed to reflect an item or notion that conceptually relates to the rest of the items under a particular factor. The reliability test is reported in Table 4.6. At this point only the initial of internal reliability of the expected factors was performed in the form of Cronbach's coefficient α. For the purposes of this study, the cut-off value adopted was 0.5 (Nunnally, 1978) and the acceptable benchmark level of item-to-total correlation was set above 0.3. Following the decision relating to internal reliability, the factors were re-specified. This was undertaken to further reduce the number of factors. The internal reliability of the first structure was tested and the decision results provide evidence as to the weakness of the structure since two factors (factors A and B) exceeded the adopted criteria. It is found that factor A contains two items and relates to the "reference group". Factor B is made up of two items and refers to "monitor investments". The derived scales appear to possess moderate to weak internal consistency. So, we eliminated factors C, D and E (see Table 4.7). To examine possible differences in the perceived importance of five factors, our analyses indicate that out of four criteria (i.e., rotated principal component loadings, scree test, KMO, and Bartlett's test, reliability test) examined, only two factors (reference group and monitor investments) are significant. The most important factor is the reference group and the second important factor is monitoring investments (see Fig. 4.1).


**Fig. 4.1. Scree plot**

**Table 4.1. The coefficient of variation (CV) of the estimates of the main items in the survey questionnaire**

| Items | | No. | % of total |
|---|---|---|---|
| 1. | When making investment decisions *today*, which of the following factors do you consider most important when making investments? Choose one alternative: (C.V = 1.91%) | | |

| **Items** | | **No.** | **% of total** |
|---|---|---|---|
| | Information from the company as a basis for a fundamental analysis. | 303 | 25.3 |
| | Recommendations, advice and forecasts from professional investors. | 221 | 18.4 |
| | The overall past performance of the market seen from a historical perspective. | 301 | 25.1 |
| | Information from newspapers / TV. | 113 | 9.4 |
| | Information from the Internet. | 47 | 3.9 |
| | Discussion with personal friends. | 85 | 7.1 |
| | Information from colleagues at work. | 30 | 2.5 |
| | Own intuition of future performance. | 99 | 8.3 |
| 2. | When you made investment decisions *during* the period from January 2006 to the end of October 2007, which of the following factors did you consider most important when making decision. Choose one alternative: (C.V. = 1.82%) | | |
| | Information from the company as a basis for a fundamental analysis. | 242 | 20.2 |
| | Recommendations, advice and forecasts from professional investors. | 265 | 22.1 |
| | The overall past performance of the market seen from a historical perspective. | 287 | 23.9 |
| | Information from newspapers / TV. | 125 | 10.4 |
| | Information from the Internet. | 58 | 4.8 |
| | Discussion with personal friends. | 89 | 7.4 |
| | Information from colleagues at work. | 38 | 3.2 |
| | Own intuition of future performance. | 95 | 7.9 |
| 3. | Do you monitor your investments with a short-term investment horizon more often today compared with the period before the market decline at the end of October 2007. Choose one alternative: (C.V. = 1.34%) | | |
| | Yes | 413 | 34.4 |
| | No | 222 | 18.5 |
| | The same | 448 | 37.4 |
| | Cannot say | 116 | 9.7 |
| 4. | Do you monitor your investments with a long-term investment horizon more often today compared with period before the market decline at the end of October 2007. Choose one alternative: (C.V. =1.26%) | | |
| | Yes | 383 | 31.9 |
| | No | 152 | 12.7 |
| | The same | 566 | 47.2 |
| | Cannot say | 96 | 8.0 |
| 5. | Please choose your relevant age group: (C.V. = 1.42%) | | |
| | 18 - 25 years old | 397 | 33.1 |
| | 26 – 35 years old | 297 | 24.8 |
| | 36 – 50 years old | 332 | 27.7 |
| | 51 – 65 years old | 148 | 12.3 |
| | over 65 years old | 25 | 2.1 |

| **Items** | | **No.** | **% of total** |
|---|---|---|---|
| 6. | Your average monthly income (including salaries, interest, rent and other earnings): (C.V. = 1.67%) | | |
| | Below HK$5,000 | 265 | 22.1 |
| | HK$5,000 -HK$9,999 | 226 | 18.8 |
| | HK$10,000 - HK$14,999 | 268 | 22.4 |
| | HK$15,000 - HK$19,999 | 193 | 16.1 |
| | HK$20,000 - HK$24,999 | 117 | 9.8 |
| | HK$25,000 - HK$29,999 | 46 | 3.8 |
| | HK$30,000 - HK$49,999 | 52 | 4.3 |
| | HK$50,000 or above | 32 | 2.7 |
| 7. | During the increases in equity prices from January 2006 up to the end of October 2007, did you at any point in time think that you could forecast the future market development? (C.V. = 1.09%) | | |
| | Yes | 336 | 28.0 |
| | No | 490 | 40.9 |
| | Cannot say | 369 | 30.8 |
| 8. | During the increases in equity prices from January 2006 up to the end of October 2007, how did you react to announcements and other information from companies? Choose one alternative: (C.V. = 1.07%) | | |
| | I made changes in my portfolio after the first news announcements | 182 | 15.2 |
| | I made changes in my portfolio after a number of consequent news announcements that pointed into the same direction | 465 | 38.8 |
| | I was not concerned about news announcements | 393 | 32.2 |
| | I cannot say | 158 | 13.2 |
| 9. | What do you think was the most important contributing factor to the *decline* in the market from the end of October 2007 up until today? Choose one alternative: (C.V. = 1.10%) | | |
| | The news stories in the media. | 120 | 10.0 |
| | The forecasts of analysts. | 95 | 7.9 |
| | Loss of confidence among investors in the stock market. | 391 | 32.6 |
| | Earnings and profitability of the listed companies. | 214 | 17.8 |
| | Herd behavior, i.e. small investors following the majority. | 294 | 24.5 |
| 10. | According to you, what is generally the reason for your less successful investments? Choose one alternative: (C.V. = 0.99%) | | |
| | Incorrect recommendations or advice from broker /analyst/ banker etc. | 151 | 12.6 |
| | Incorrect recommendations or advice from other sources | 161 | 13.4 |
| | The market has, in general, performed poorly | 460 | 38.4 |
| | Own errors | 404 | 33.7 |
| | Others (please specify):____________________________ | 22 | 1.8 |

**Table 4.2. Descriptive statistics**

| Item | Item name | Mean | Standard error of mean | Standard deviation | t | d.f. | Sig. (two-tailed) |
|---|---|---|---|---|---|---|---|
| 1 | Reference group affects investment decision today | 3.2085 | 0.06132 | 2.12346 | 52.320 | 1198 | 0.000 |
| 2 | Reference group affected past investment decision | 3.3219 | 0.06045 | 2.09334 | 54.949 | 1198 | 0.000 |
| 3 | Monitor short-term investments | 2.2227 | 0.02968 | 1.02780 | 74.882 | 1198 | 0.000 |
| 4 | Monitor long-term investments | 2.3133 | 0.02914 | 1.00813 | 79.389 | 1196 | 0.000 |
| 5 | Age | 2.2552 | 0.03197 | 1.10693 | 70.547 | 1198 | 0.000 |
| 6 | Personal income | 3.1476 | 0.05255 | 1.81968 | 59.896 | 1198 | 0.000 |
| 7 | Forecasting the future market development | 2.0276 | 0.02221 | 0.76791 | 91.276 | 1194 | 0.000 |
| 8 | Announcements from companies | 2.4399 | 0.02608 | 0.90260 | 93.564 | 1197 | 0.000 |
| 9 | Factor for bear market | 3.4192 | 0.03777 | 1.26079 | 90.516 | 1113 | 0.000 |
| 10 | Reason for investment failure | 2.9875 | 0.02960 | 1.02468 | 100.913 | 1197 | 0.000 |

### Table 4.3. Factor correlation matrix

| Item | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | 0.615** | 1.000 | | | | | | | | |
| 3 | 0.067* | 0.035 | 1.000 | | | | | | | |
| 4 | 0.045 | 0.045 | 0.444** | 1.000 | | | | | | |
| 5 | 0.062* | 0.057* | -0.014 | -0.047 | 1.000 | | | | | |
| 6 | -0.043 | -0.020 | -0.060* | -0.036 | 0.315** | 1.000 | | | | |
| 7 | -0.002 | 0.022 | 0.104** | 0.081** | 0.002 | -0.089** | 1.000 | | | |
| 8 | 0.120** | 0.092** | 0.257** | 0.195** | -0.023 | -0.085** | 0.206** | 1.000 | | |
| 9 | -0.009 | 0.012 | -0.025 | 0.049 | -0.031 | 0.049 | 0.023 | -0.020 | 1.000 | |
| 10 | 0.032 | 0.054* | 0.055* | 0.087** | -0.066* | 0.058* | 0.071** | 0.059* | 0.021 | 1.000 |

*Notes: *Correlation is significant at the 0.05 level (one-tailed) and **Correlation is significant at the 0.01 level (one-tailed)*
*Extraction method: principal component analysis, Rotation method: Varimax with Kariser Normalization,*
*Kaiser-Meyer-Olkin (KMO) index: 0.546, Bartlett's test of Sphericity: ρ<0.000.*
*Item name (see also Table 4.3) 1.Reference group affects investment decision today, 2. Reference group affected past investment decision, 3.Monitor short-term investments, 4. Monitor long-term investments, 5.Age, 6. Personal income, 7. Forecasting the future market development, 8. Announcements from companies, 9. Factor for bear market, 10.Reason for investment failure.*

**Table 4.4. Principal component analysis**

| Item | Item name | Communality | Factor (Component) | Eigenvalue | Per cent of variance | Cumulative per cent |
|---|---|---|---|---|---|---|
| 1 | Reference group affects investment decision today | 0.813 | 1 | 1.877 | 18.768 | 18.768 |
| 2 | Reference group affected past investment decision | 0.811 | 2 | 1.545 | 15.451 | 34.219 |
| 3 | Monitor short-term investments | 0.716 | 3 | 1.268 | 12.678 | 46.897 |
| 4 | Monitor long-term investments | 0.704 | 4 | 1.052 | 10.520 | 57.417 |
| 5 | Age | 0.720 | 5 | 1.013 | 10.130 | 67.547 |
| 6 | Personal income | 0.700 | | | | |
| 7 | Forecasting the future market development | 0.786 | | | | |
| 8 | Announcements from companies | 0.513 | | | | |
| 9 | Factor for bear market | 0.534 | | | | |
| 10 | Reason for investment failure | 0.459 | | | | |

### Table 4.5. Varimax-rotated principal component loadings

| | Factor | | | | | | |
|---|---|---|---|---|---|---|---|
| **Item** | **A** | **B** | **C** | **D** | **E** | **Item name** | **Factor** |
| 1 | 0.900 | | | | | Reference group affects investment decision today | A |
| 2 | 0.898 | | | | | Reference group affected past investment decision | A |
| 3 | | 0.836 | | | | Monitor short-term investments | B |
| 4 | | 0.828 | | | | Monitor long-term investments | B |
| 5 | | | 0.817 | | | Age | C |
| 6 | | | 0.799 | | | Personal income | C |
| 7 | | | | 0.877 | | Forecasting the future market development | D |
| 8 | | | | 0.594 | | Announcements from companies | D |
| 9 | | | | | 0.722 | Factor for bear market | E |
| 10 | | | | | 0.651 | Reason for investment failure | E |

**Table 4.6. Internal consistency and related decisions of first structure**

| Factors and items | Corrected item-total correlation | α value | Decision |
|---|---|---|---|
| Factor A (Reference Group) | | | |
| Reference group affects investment decision today | 0.6155 | 0.7619 | Retained |
| Reference group affected past investment decision | 0.6155 | | |
| Factor B (Monitor Investments) | | | |
| Monitor short-term investments | 0.4436 | 0.6145 | Retained |
| Monitor long-term investments | 0.4436 | | |
| Factor C (Personal Background) | | | |
| Age | 0.3149 | 0.4370 | Eliminated |
| Personal income | 0.3149 | | |
| Factor D (Reaction to announcements) | | | |
| Forecasting the future market development | 0.2060 | 0.3380 | Eliminated |
| Announcements from companies | 0.2060 | | |
| Factor E (Cognitive Style) | | | |
| Factor for bear market | 0.0214 | 0.0410 | Eliminated |
| Reason for investment failure | 0.0214 | | |

**Table 4.7. Internal consistency of final revised structure**

| Items | Number of item | Corrected item-total correlation | α value |
|---|---|---|---|
| Factor A (Reference Group) | | | |
| Reference group affects investment decision today | 2 | 0.6155 | 0.7619 |
| Reference group affected past investment decision | | 0.6155 | |
| Factor B (Monitor Investments) | | | |
| Monitor short-term investments | 2 | 0.4436 | 0.6145 |
| Monitor long-term investments | | 0.4436 | |

## 4.6 CONCLUSION

Using factor analysis, we identify five factors that capture the behaviour of small investors in the Hong Kong stock market. The factors are reference group, monitor investments, personal background, reaction to announcements and cognitive style. The factor of reference group includes commentators' recommendations from newspapers/TV/magazines, relatives/friends, Internet, investment consultants, and companies' annual reports; the factor of monitor investments includes monitoring short-term and long-term investments; the factor of personal background includes age and personal income; the factor of reaction to announcements includes announcements and other information from companies, forecasting the future market development and the factor of cognitive style includes factor for bear market and reason for investment failure. In order to examine possible differences in the perceived importance of the five factors, our analysis indicates that out of four criteria (including rotated minimum residual solution, scree test, KMO and Bartlett's test, and reliability test) examined, only two factors (i.e., reference group, monitor investments) stand out to be significant. Accordingly, two factors can affect the investment behaviour of small investors in the Hong Kong stock market. The most important factor is the reference group and the second important factor is monitor investments. Although the present study is exploratory in nature, some new results are obtained that are in line with the predictions of behavioural finance.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

## REFERENCES

Arisoy, Y. E., Bali, T. G., and Tang,Y..2024. Investor Regret and Stock Returns. Management Science, Articles in Advance, 1–22.

Bell, D.E. (1982) Regret in Decision Making under Uncertainty. Operations Research, 30, 961-981.
http://dx.doi.org/10.1287/opre.30.5.961

Cattell, R. B.,1966. The Meaning and Strategic Use of Factor Analysis. Handbook of Multivariate Experimental Psychology, Chicago: Rand-McNally.

Fama, E. F. and MacBeth, J. D., Risk, Return, and Equilibrium: Empirical Tests. Journal of Political Economy. Volume 81, Number 3. https://doi.org/10.1086/260061

Hon T.Y., 2012. The Behaviour of Small Investor in the Hong Kong Stock Market. Asian Profile, 40(3), 225-236.

Hon T.Y., 2015. A Factor Analysis of Investment Behaviour for Small Investors in the Hong Kong Stock Market. Journal of Economics Library, Volume 2, Issue 2, 68-78.

Johnsson, M., H. Lindblom, and P. Platan, 2002. Behavioral Finance- And the Change of Investor Behavior During and after the Speculative Bubble at the End of the 1990s. Master Thesis, School of Economics and Management of Lund University, Sweden.

Li, J. and Ahn, H. J. 2024. Sensitivity of Chinese stock markets to individual investor sentiment: An analysis of Sina Weibo mood related to COVID-19. Journal of Behavioral and Experimental Finance, Volume 41.
Loomes, G. and Sugden, R. 1982. Regret Theory: An Alternative Theory of Rational Choice Under Uncertainty. The Economic Journal, Vol. 92, No. 368, 805-824.
Morgan, D. L. 2008. The SAGE Encyclopedia of Qualitative Research Methods. SAGE publications, Inc., 816-817.
Nunnally, J. C., 1978. Psychometric Theory (2$^{nd}$ed.). New York: McGraw-Hill.
Thurstone, L.L., 1947. Multiple Factor Analysis. Chicago: University of Chicago Press.
Varshini, R. and Vinayalaxmi. 2024. An Analysis of Investor behaviour in the Equity Market Dynamics. EPRA International Journal of Economic and Business Review, Volume 12, Issue 1, 31-36.
DOI: https://doi.org/10.36713/epra15682
Xia, Y. and Madni,G. R., 2024. Unleashing the behavioral factors affecting the decision making of Chinese investors in stock markets. PLOS ONE, 19(2), 1-18.

**Biography of author(s)**

**Tai-Yuen Hon (Research Affiliate)**
Business, Economic and Public Policy Research Centre, Hong Kong Shue Yan University, Hong Kong.

He was an Academic Assistant, Lecturer, Senior Lecturer, and Assistant Professor in the Department of Economics and Finance (formerly known as the Department of Economics) of Hong Kong Shue Yan University from 1993 to 2016 and is a Research Affiliate at the Business, Economic and Public Policy Research Centre of this University. He obtained his PhD in Business Administration from Bulacan State University, and Master of Arts in Money, Banking and Finance from the University of Sheffield. He has published papers in the Journal of Risk and Financial Management, Journal of Family and Economic Issues, International Journal of Revenue Management, Advances in Decision Sciences, and others.

**Wing-Kwong Au (Associate Professor)**
Department of Social Work, Hong Kong Shue Yan University, Hong Kong.

He has been working at the Department of Social Work of Hong Kong Shue Yan University since 1990. He is an Associate Professor and Director of the China Liaison Office of the University, a Fellow of the University of Liverpool and a Registered Social Worker of Social Workers Registration Board, Hong Kong. He obtained his MPhil and Ph. D from the University of Liverpool and Master of Arts in Social Work from the University of Wales, Bangor (Bangor University at present) in the United Kingdom. His research interests include elderly services; community development; social services in China (PRC); children and marginal youth; youth work and behavioural science and finance. He has published reports on Hidden Youth Drug Abusers; Die in Exhaustion and the Labour Compensation in Hong Kong; the Interventions of the Marginal Youth Behavioural Pattern; Positive Life of Young People in Eastern District of Hong Kong, and a conference paper in Young Night Drifters' Social Workers and Health. He has also published two books "Eastern District Positive Life project 15th Anniversary: A Book of Reflections" and "The Interventions of the Marginal Youth Behavioural Pattern". A survey report on "Children's life living in sub-divided flats under COVID-19" is forthcoming. His current project is a book chapter of "Review on Market Efficiency and Anomalies" and "The Dilemma of Investment Decision for Small Investors in the Hong Kong Stock Market". A book chapter "The Development of Social Welfare and Services of Hong Kong since the Colonization" in the book of Hong Kong Literature Studies, Hong Kong and Macau Study Centre Jinan University, China, China: Jinan University Press, forthcoming.

**Wing-Keung Wong (Chair Professor)**
Department of Finance, Fintech & Blockchain Research Center, and Big Data Research Center, Asia University, Taiwan.
Department of Medical Research, China Medical University Hospital Business, Taiwan.
Economic and Public Policy Research Centre, Hong Kong Shue Yan University, Hong Kong.

He obtained his Ph.D. from the University of Wisconsin-Madison, the USA with a major in Business Statistics (Statistics and Finance) and obtained his Bachelor's degree from the Chinese University of Hong Kong, Hong Kong, with a major in Mathematics and a double minor in Economics and Statistics. Currently, he is a Chair Professor at the Department of Finance, at Asia University. He was a Full Professor at the Department of Economics, Hong Kong Baptist University, and Deputy Director at the Risk Management Institute, National University of Singapore.

He appears in "Who's Who in the World" and gets the Albert Nelson Marquis Lifetime Achievement Award. 2017, Marquis Who's Who, and is on the list of the "2023 World's top 2% of researchers/scientists", on the list of the "Best Economics and Finance Scientists", ranked 788th in the world and ranked first in Taiwan with a D-index of 52, and on the list of the 2023 Asia Scientists Economics & Econometrics Rankings, ranked first in Taiwan with an H index of 66, i10 index of 281, and citations of 14925. His Erdos number is 3. He is ranked top 1% by the Social Science Research Network and in the list of top Taiwan economists, top Asian economists, and top world economists by RePEc. He has published more than four hundred papers including papers published in some top journals. He has more than 17000 citations in Google Scholar, more than 15000 citations in Researchgate, and more than 16000 citations in Scopus. His h-index is 69, (46 since 2019) and his i10-index is 314, (278 since 2019) by Google Scholar citation in March 2024.

He is on the list of top (1st, 0.384%) Taiwan economists, top (1st, 0.384%) Taiwan economists (counted publications last 10 years), top (39th, 0.48%) Asian economists (counted publications last 10 years), top (39th, 0.48%) Asian economists, (417th, 0.62%) [World] authors [in Economics in last 10 years] and (614th, 0.913%) [World] authors [in Economics], top (199th, 0.296%) in Number of Works, top (117th, 0.174%) in Number of Distinct Works, top (682nd, 1.01%) in Number of Distinct Works, Weighted by Number of Authors, top (37th, 0.055%) in Number of Journal Pages, top (201st, 0.299%) in Number of Journal Pages, Weighted by Number of Authors, top (900th, 1.34%) in Record of graduates, top (47th, 0.0699%) in Closeness measure in the co-authorship network, top (10th, 0.0149%) in Betweenness measure in the co-authorship network by, etc. by RePEc in December 2023. I have 37 items ranked within 15%, 32 items ranked within 10%, 22 items ranked within 5%, 18 items ranked within 3%, 18 items ranked within 2%, and 14 items within 1% among all Economists registered in RePEc in December 2023, etc.

He has been serving international academies, Government, society, and universities, providing consultancy to several Government departments and corporations, and giving lectures and seminars to several universities. For example, he has been serving as editor, guest leading editor, advisor, associate editor for some international journals, appointed as an advisor/member of various international associations/institutes, serving as a referee for many journals/conferences, supervising solely or jointly several overseas graduate students, appointed as an external reviewer and external examiner by other universities, and invited by many universities/institutions to present papers or conduct seminars, for example, he is the Editor-in-Chief of Annals of Financial Economics, a Senior Co-Editor-in-Chief and Managing Editor for Advances in Decision Sciences, an Editor for Risk Management, Cogent Economics and Finance, Journal of Risk and Financial Management, Mathematical Problems in Engineering; Advisor for Asian Academy of Management Journal of Accounting and Finance; Associate Editor for International Journal of Emerging Markets, Financial Innovation, International Journal of Energy Research, Heliyon, Communications in Statistics: Theory and Methods, Communications in Statistics: Simulation and Computation, Asia-Pacific Journal of Operational Research, Studies in Economics and Finance, Economies, Journal of Infrastructure, Policy and Development, Communications in Statistics: Case Studies, Data Analysis and Applications, etc.

He has published more than four hundred papers including papers published in journals ranked as A* in ABDC, Q1 in SJR Quartile, Q1 in JCR, and 4 in AJG and including papers published in Journal of Finance, Contemporary Accounting Research, Annals of Applied Probability, Scientific Report, Mathematical Finance, European Journal of Operational Research, Journal of Business and Economic Statistics, Economic Theory, Journal of Empirical Finance, Journal of Financial Markets, Journal of Economic Behavior and Organization, Economics Letters, Econometrics Journal, Quantitative Finance, Economic Inquiry, Energy Economics, Statistics and Probability Letters, Journal of Risk, Journal of Operational Research Society, Journal of Financial Econometrics, Journal of Forecasting, Journal of International Financial Markets, Institutions & Money, Transport Policy, Personality and Individual Differences, Journal of Time Series Analysis, Applied Economics, Journal of Multinational Financial Management, Journal of Behavioral Finance, Pacific-Basin Finance Journal, Global Finance Journal, Annals of Finance, Mathematics and Computers in Simulation, IMA Journal of Management Mathematics, Review of Quantitative Finance and Accounting, North American Journal of Economics and Finance, Accounting & Finance, Economic Modelling, Energy Policy, Applied Mathematics Letters, Journal of International Consumer Marketing, Statistical Papers, Japanese Economic Review, Global Economic Review, Multinational Finance Journal, Central European Journal of Mathematics, Journal of Asian Economics, Mathematical Scientist, Journal of Retailing and Consumer Services, International Review of Financial Analysis, Current Issues in Method and Practice, Sustainability, Risk Management, Scientific Reports, International Review of Economics & Finance, Studies in Nonlinear Dynamics and Econometrics, Current Issues in Tourism, Finance Research Letters, Journal of the Korean Statistical Society, PLOS ONE, Journal of Testing and Evaluation, Estudios de Economía, Physica A: Statistical Mechanics and its Applications, International Journal of Production Research, Resources Policy, International Journal of Environmental Research and Public Health, World Economy, Emerging Markets Review, Energies, Economies, Asia Pacific Journal of Operational Research, Review of Pacific Basin Financial Markets and Policies, International Journal of Financial Engineering, Emerging Markets Review, Journal of Theoretical and Applied Electronic Commerce Research, Asian Academy of Management Journal of Accounting & Finance, Annals of Financial Economics, Asia-Pacific Journal of Business Administration, Econometrics and Statistics, Communications in Statistics: Case Studies, Data Analysis and Applications, Communications in Statistics: Theory and Methods, Communications in Statistics: Simulation and Computation, Research in International Business and Finance, Journal of Risk and Financial Management, Energy Reports, Frontiers in Public Health, Environmental Science and Pollution Research, Kybernetes, Journal of Asian Business and Economic Studies, International Journal of Emerging Markets, Emerging Markets Finance and Trade, Fuel, Mathematical and Computational Applications, Cogent Economics and Finance, Asia-Pacific Journal of Operational Research, Chaos, Solitons & Fractals, Frontiers in Public Health-Health Economics, Fractals, Computers and Operations Research, Economic Research-Ekonomska Istraživanj, Democracy and Security, Heliyon, Journal of International Commerce, Economics and Policy, etc.

---



**DISCLAIMER**
This chapter is an extended version of the article published by the same author(s) in the following journal.
Journal of Risk and Financial Management, 5: 59-77, 2012. Available: https://www.mdpi.com/1911-8074/5
Journal of Economics Library, 2(2): 69-78, 2015. Available: http://kspjournals.org/index.php/JEL/article/view/258

**Peer-Review History:**
This chapter was reviewed by following the Advanced Open Peer Review policy. This chapter was thoroughly checked to prevent plagiarism. As per editorial policy, a minimum of two peer-reviewers reviewed the manuscript. After review and revision of the manuscript, the Book Editor approved the manuscript for final publication. Peer review comments, comments of the editor(s), etc. are available here: https://peerreviewarchive.com/review-history/12411F-CH4

**Chapter 5**
Print ISBN: 978-81-973195-8-7, eBook ISBN: 978-81-973195-0-1

# Analysing the Impact of Economic Variables on Hang Seng Sub-Indexes Performance in Hong Kong

**Hok-Fu Wu [a] and Kai-Yin Woo [b++*]**

DOI: 10.9734/bpi/mono/978-81-973195-8-7/CH5

**Peer-Review History:**
This chapter was reviewed by following the Advanced Open Peer Review policy. This chapter was thoroughly checked to prevent plagiarism. As per editorial policy, a minimum of two peer-reviewers reviewed the manuscript. After review and revision of the manuscript, the Book Editor approved the manuscript for final publication. Peer review comments, comments of the editor(s), etc. are available here: https://peerreviewarchive.com/review-history/12411F-CH5

## ABSTRACT

In the literature, the relationship between stock price and economic variables was examined in different countries. The findings confirmed the existence of fundamental variables in the Hong Kong stock market. This paper focuses on the case of Hong Kong and extends this issue to the Hang Seng sub-indexes, which include the Commerce and Industry Sub-index, the Finance Sub-index, the Properties Sub-index and the Utilities Sub-index, by applying the Vector Error Correction Model based on the sample from January 2004 to August 2014. Applying Johansen's (1991) method, cointegration is found between each of the sub-indexes and different sets of economic variables, including price level, money supply, effective exchange rate, long-term interest rate, China stock market and industry-related variables. The results show that the long-run coefficients of some economic variables vary in size and sign in the cointegrating vectors in different sub-index models. Granger causality results conclude that all four sub-indexes are long-run Granger-caused by the economic variables with different speeds of adjustment. The paper also finds that industry-specific variables, relative to the macroeconomic fundamentals, are playing modest roles in determining the long-run equilibrium of the stock indexes. In addition, impulse responses and variance decomposition analysis are performed to show the relative strength of the causal chain between the sub-indexes and economic variables. This paper could draw implications for investors in their decision-making process about how the stock performance in various sectors is affected by different economic fundamentals.

---

*[a] Crowe (HK) Global Corporate Advisory Limited, Hong Kong.*
*[b] Department of Economics and Finance, Hong Kong Shue Yan University, Hong Kong.*
*[++] Associate Professor;*
**Corresponding author: E-mail: kywoo@hksyu.edu;*

*Keywords: Hang Seng sub-indexes; cointegration; causality; impulse response; variance decomposition.*

**JEL classifications:** E4, G1.

## 5.1 INTRODUCTION

The connection between stock market performance and economic condition has been widely accepted in the realm of financial economics. Many literatures show that the stock market plays an essential part in economic development. A lot of theories developed over the past decades have laid the foundation for financial economics. Nowadays, modern empirical studies apply the time-series econometric methods to further investigate the interactions between stock markets and the real economy. Not only do these results confirm predecessors' ideas, but they also facilitate our understanding of that interrelationship in reality. Yet, further research in this topic is still needed in order to uncover unexplored areas in the field and adapt to the ever-changing economic environment in aspects such as globalization and institutional changes.

This paper aims to study the interactions between the Hong Kong stock market and a bundle of macroeconomic variables. In fact, there are pieces of literature examining similar topics. Patra and Poshakwale (2006) looked into the long-run and short-run relationship between economic variables and stock market return in Athens. Gjerde and Sattem (1999) inspected the stock returns in the Norwegian economy. Montes and Tiberto (2012) studied the case in Brazil. Kwon and Shin (1999) assessed the cointegration and Granger causality in Korean stock market returns. However, few have researched the case of Hong Kong. Lai et al. (2013) once compared the dynamic interaction between macroeconomic variables and stock indexes in China, Taiwan and Hong Kong, using monthly data from 1991 to 2008. Their findings give a hint that there might be interactions among local variables and the stock market in Hong Kong.

In previous studies, there has been a plentiful focus on the general stock market index like the S&P 500, Hang Seng Index and NIKKEI225, while neglecting how sector stock indexes interact with the economy. Nevertheless, these sub-indexes are increasingly important; a number of researches investigated the implication of the sub-indexes in the equity market. Gondhalekar and Mehdian (2003) explored the Monday effect on the NASDAQ composite index and its sub-indexes. Patra and Poshakwale (2008) investigated the short-term and long-term relationships among sub-indexes in the case of the ATHEN stock exchange. As a matter of fact, the Hang Seng sub-indexes in Hong Kong were recently studied with the aim of examining the causality of bubbles; their findings depict the interactive role that each sector is taking in causing bubbles (Miyakoshi et al., 2014).

Unlike the majority of the studies, this paper takes one step forward by investigating the stock sub-indexes in Hong Kong, which are compiled by categorizing the parent index into different sectors. It intends to delve into the causal relationship between macroeconomic variables and the sub-indexes. Stocks in different sectors are often influenced by variables from three levels, namely the macroeconomic factors, the industry-specific factors and firm-specific factors. As our interest is in stock market indexes, not individual stocks, firm-specific factors are not a concern in this paper. One sector's index could demonstrate very distinctive characteristics from another. Under this circumstance, how Hang Seng sub-indexes are affected by macroeconomic variables and industry-specific variables are the focal points here.

The start of the Hang Seng Index (HSI) in Hong Kong dates back to 1969, but the sub-indexes were established later by dividing the constituents into four groups. They now refer to the Hang Seng Commerce & Industry Sub-index (C&I), the Hang Seng Finance Sub-index (FIN), the Hang Seng Properties Sub-index (PROP) and the Hang Seng Utilities Sub-index (UTI). Table 5.1 shows the industry-based classification system for the sub-indexes.

**Table 5.1. Mapping table of Hang Seng industry classification system**

| **Hang Seng Sub-Indexes** | **Industry** |
|---|---|
| Finance | Financials |
| Utilities | Utilities |
| Properties | Properties & Construction |
| Commerce and Industries | Energy |
| | Materials |
| | Industrial Goods |
| | Consumer Goods |
| | Services |
| | Telecommunications |
| | Information Technology |
| | Conglomerates |

*Source: Hang Seng Indexes Company (http://www.hsi.com.hk)*

There are primarily two reasons for studying Hang Seng sub-indexes. Firstly, the Hong Kong stock market is regarded as the financial hub that bridges the financing activities between China and the rest of the world. In early 1990, China's enterprises decided to issue the so-called H-shares in Hong Kong as they found it hard to attract foreign investment by issuing B-shares on the Shanghai Stock Exchange and Shenzhen Stock Exchange (Nie, 1997). After the sovereignty transfer of Hong Kong to China in 1997, more Chinese enterprises preferred listing in Hong Kong to finance their businesses, for example, China Unicom in 2000, China Shenhua Energy in 2005 and the Bank of China in 2006. Table 5.2 shows the constituents of Hang Seng Sub-indexes; among these 50 constituents, 19 of them are either the constituents of the Hang Seng China Enterprises Index (HSCEI) or the Hang Seng China-Affiliated Corporations Index (HSCCI). Second, the Hong Kong stock market has been undergoing dramatic changes. The manufacturing sector in Hong Kong has declined since 1980. The finance sector was severely impacted by the financial crises. The property sector has attracted a lot of investment from Mainlanders. Different sectors or industries have been exposed to different kinds of shocks and influences over time; the sub-sector stock indexes thus respond differently. Fig. 5.1 shows the movement of the Hang Seng Sub-indexes over the past ten years. It can be observed that the trend of UTI was relatively stable and was growing faster than other sub-indexes with a gloomy economic outlook on average. The C&I is the most stable one among the four sub-indexes. The FIN and PROP are more volatile and appear to be moving together. These characteristics show that it would be useful to understand how the Hang Seng sub-indexes have responded differently to macroeconomic and industry factors in recent years.

The remainder of the paper proceeds as follows. Section 2 reviews the relevant literature. Sections 3 and 4 outline the model specifications and the econometric methodology. Section 5 describes the data. The empirical results are presented in Section 6, and conclusions are stated in the last section. The influence of HSCEI and HSCCI on the HSI is shown in the appendix.

**Table 5.2. Constituents of the Hang Seng Sub-indexes**

| **Commerce and Industry Sub-index** | | | | **Finance Sub-index** | |
|---|---|---|---|---|---|
| Hutchison | [Jan78] | Li & Fung | [Aug00] | HSBC | [Apr91] |
| Swire Pacific A' | [Jul64] | Tencent | [Jun08] | Hang Seng Bank | [Jul72] |
| Galaxy Ent | [Jun13] | China Unicom | (R)[Jun01] | Bank of E Asia | [Jan84] |
| MTR Corporation | [Jun01] | PetroChina | (H)[Dec07] | HKEx | [Sep06] |
| Kunlun Energy | (R)[Dec12] | CNNOC | [Jul01] | CCB | (H)[Sep06] |
| China Mer Hldings | (R)[Sep04] | China Mobile | (R)[Jan98] | AIA | [Jun11] |
| Want Want China | [Dec11] | Lenovo Group | (R)*[Aug00] | ICBC | (H)[Mar07] |
| CITIC | (R)[Aug92] | Hengan Int'l | [Jun11] | Ping An | (H)[Jun07] |
| China Resources | (R)[Jul97] | China Shenhua | (H)[Dec07] | BOC Hong Kong | [Dec02] |
| Cathay Pac Air | [Jun86] | Belle Int'l | [Sep10] | China Life | (H)[Mar07] |
| Tingyi | [Dec11] | Sands China | [Jun12]] | Bankcomm | (H)[Sep07] |
| Sinopec Corp | (H)[Dec06] | Mengniu Dairy | [Mar14] | Bank of China | (H)[Dec06] |
| **Properties Sub-index** | | **Utilities Sub-index** | | * Wharf (Hlgs): added in Oct86, reclassified into PROP in Sep12. New World Dev: added in Mar73, deleted in Jun03, added in Jun05, reclassified into PROP in Sep 12. Sino Land: added in | |
| Cheung Kong | [Jan78] | CLP Holdings | [Jan98] | | |
| Wharf (Hldgs) | *[Oct86] | HK & China Gas | [Jul64] | | |
| Henderson Land | [Jan84] | Power Assets | [Jul64] | | |
| SHK Prop | [Dec78] | China Res Power | (R)[Jun09] | | |

| | | |
|---|---|---|
| New World Dev | *[Oct73] | Feb95, deleted in Jun03 and added again in Jun05 |
| Sino Land | *[Feb95] | |
| Hang Lung Prop | [Nov94] | Lenovo: added in Aug00, deleted in Sep06, added again in Mar13. |
| China Overseas | (R)[Dec07] | |
| Link REIT | [Dec14] | |
| China Res Land | (R)[Mar10] | |

Notes:
The month and year of being first included in HSI in [].
(H) and (R) denote the current constitutes of the Hang Seng China Enterprises Index and Hang Seng China-Affiliated Corporations Index.
Source: Hang Seng Indexes Company Limited

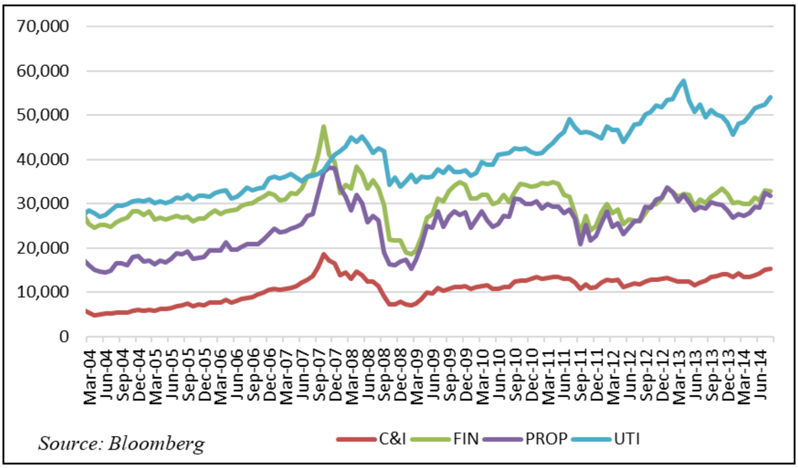



**Fig. 5.1. Hang Seng Sub-indexes**

## 5.2 LITERATURE REVIEW

The Hong Kong Stock Exchange is one of the world's major stock exchanges; in 2011 it was the third-largest stock exchange by market capitalization in Asia. As companies choose to list in Hong Kong, they intend to attract capital from the neighborhood for financing their business. For this reason, the local economy is decisive to the stock price regardless of the companies' country of origin.

In theory, economic factors like price level, money supply and nominal exchange rate all relate to the stock market in certain ways. Many previous studies included these three factors in their models (Gjerde & Saettem, 1999; Montes &Tiberto, 2012; Patra & Poshakwale, 2006). Thus, here we will pay particular attention to these variables.

The traditional economic cycle inclines to give people the impression that stock price positively correlates to inflation. When the economy is at the growing stage, inflation rises, and the stock market goes up. A correlation matrix between stock price and inflation can easily generate this positive relationship. Nevertheless, this is not the whole story. Previous empirical studies found that the stock prices and the general price level go in opposite directions (Cohn & Lessard, 1981; Fama 1981). In the money demand theory and the quantity theory of money, an increase in price discourages real economic activity, other things being equal. Fama (1981) pointed out that the negative stock return-inflation is caused by the proxy effect from the positive relationship between real economic activity and real stock return. Money supply is another variable that can significantly affect the investment portfolio and hence the stock price. Money demand theory and the discount model give a clue as to how stock price changes with money supply (Hamburger & Kochin, 1972; Homa & Jaffee, 1972). It can be shown that by these two theories, there are several ways the money supply exerts influence on equity price, including the liquidity effect, earning effect and risk premium effect. Typical discounting models can demonstrate the three effects just mentioned; the asset price valuation implies the positive relationship between price and future cash flow, and the negative relationship between price and discount rates. To illustrate the idea, the discounted cash flow model for the present value (PV) of stock price at time t is simplified, by assuming $r_f$ and $r_p$ to be constant over time, as follows,

$$PV_t = \sum_{i=1}^{\infty} \frac{CF_{t+i}}{\left(1 + r_f + r_p\right)^i} \tag{1}$$

where $CF$ is the free cash flow generated by the company, $r_f$ is the risk-free rate, $r_p$ is the risk premium. The liquidity effect and the earnings effect caused by an increase in money supply will consistently result in the $PV_t$ moving in the same direction; whereas the increase in risk premium induced by the money supply fluctuation will impose a negative effect on stock price.

As far as the exchange rate is concerned, this can affect stock price in mainly two ways, the “flow-approach” and the “stock-approach” (Alagidede et.al., 2011; Lee et al., 2011). The Dornbusch Model is one of the flow-oriented models; the exchange rate takes part in the determining equilibrium of the money market and goods market (Dornbusch & Fischer 1980). Exchange rate movement affects the term of trade, which would be reflected in the stock market. It is suggested that domestic currency depreciation strengthens the competitiveness of domestic exports, resulting in stock price increases. Companies with foreign operations, like most of the companies in the Hang Seng Index can be affected by the exchange rate fluctuation as earnings and costs vary accordingly (Aggarwal 1981). The stock-approach highlights the role of financial assets in the determination of exchange rates. The portfolio balance models (Branson, 1981) posit a positive relationship between the values of domestic currency and stock prices, based on the idea that when the return of domestic-issued assets is higher than that of foreign-issued assets, it attracts capital to invest in domestic assets. As a result, the demand for equity will drive up the value of domestic currency.

In the past, empirical researches which study the relationship between stock price and the economy have been done on the basis of various stock markets. They found significant relationships between macroeconomic variables and stock market return. Gjerde and Saettem (1999) took oil price changes and changes in industrial production and international industrial production into account in their model formulation. In their study of the Norwegian stock market, the oil price was reflected by the stock market as Norway heavily relied on oil. Industrial production also positively changed with stock returns. Patra and Poshakwale (2006) found long-run and short-run relationships between stock market returns in the Athens stock exchange and economic variables in the period of 1990 to 1999. By use of Granger causality testing, inflation, money supply and trading volume Granger caused the stock index in the short run. Further cointegration analysis, namely both the Engle-Granger method and the Johansen-Juselius approach, confirmed the long-run relationship among stock returns, inflation, money supply and trading volume. Montes and Tiberto (2012) included economic growth rate, market capitalization, real interest rate, exchange rate, country risk and the U.S. stock index in their model of the Brazilian stock market. The OLS estimates indicate that all these variables, except the foreign stock market, are significant and match economic theories. This highlights the importance of the role of economic policy towards the stock market performance and implies that Brazil's stock performance was highly dependent on local factors instead of being affected by stock markets in North America.

With regard to the Hong Kong case, similar studies are uncommon. Lai et al. (2013) examined the cointegrated relationship and causality directions between macroeconomic variables and stock market indexes in Hong Kong (Hang Seng Index), as well as Taiwan (Taiwan Stock Exchange Weighted Index), and China (Shanghai Stock Exchange Composite Index) based on monthly data from 1991 to 2008. Hang Seng Index is a broad market index in Hong Kong, covering companies of different industries, while Hang Seng sub-indexes cover companies of specified groups of industries. We fill the research gap by studying each of the Hang Seng sub-indexes in order to investigate more deeply how each specified industry in Hong Kong is affected by macroeconomic factors in different ways.

## 5.3 MODEL SPECIFICATION

The formulation of models to measure the relations between stock market sub-indexes and economic variables in the case of Hong Kong includes several macroeconomic variables: consumer price index (CPI), money supply (MS), effective exchange rate (HKEER), Interest rate (INT), Shanghai Stock Exchange A Share Index (SHA) and Standard and Poor's 500 Index (SPX), as shown below.

$$f(\underset{-}{CPI},\ \underset{+}{MS},\ \underset{+}{HKEER},\ \underset{-}{INT},\ \underset{+}{SHA},\ \underset{+}{SPX}) \qquad (2)$$

CPI, MS, HKEER and INT are important indicators of the Hong Kong economic and capital market condition. Based on the reasoning in the literature review and previous studies, the expectation of the relationship between explanatory variables and the sub-indexes is marked below Equation (2).

It is expected that the CPI will go in the opposite direction to the stock price. Fama (1981) pointed out that the negative relation between real stock return and inflation can be attributed to inflation's erosion on real activity and loss in real stock return due to languishing real activity outlook. Since the equity price valuation model relates to future cash flows, the movements of CPI could change the prediction of future cash flow and thus the stock price valuation.

MS should change with the stock price in the same direction. This can be illustrated by Equation (2); the MS can affect the stock valuation through $CF$ and ($r_f + r_p$) (Hamburger & Kochin 1972). A rise in MS boosts demand and indirectly benefits corporate earnings, which can be observed through an increase in $CF$ through the earnings effect. In addition, increments of MS tend to lower the interest rate ($r_f + r_p$) to reflect that the liquidity in the economy is at a state of ease as illustrated by the money demand theory. Under the linked exchange rate system in Hong Kong, this negative money-interest relationship might be weakened as the short-term rates in Hong Kong are adjusted in accordance with the U.S. Federal Funds target rates. Nevertheless, increases in money supply could still bring down the long-term interest rates as a cost of capital for companies due to the liquidity effect. Companies can negotiate better long-term costs for funding their businesses with financial institutions. For the risk premium effect, the stock price variation initiated by the MS change, regardless of positive or negative shocks, might increase $r_P$ as risk-averse investors require compensation for uncertain fluctuation of $PV_t$ or stock price. The risk premium effects on the sub-indexes should be minor because our research subject focuses on Hang Seng sub-indexes, in which most of the constituents are qualified blue chips. So, it is expected that an increase in MS will boost the sub-indexes, mostly through the earnings effect mentioned above.

HKEER is expected to have a positive relation with stock index changes. HKEER measures the value of the Hong Kong Dollar (HKD) in foreign currency; an HKEER increase implies HKD appreciation. Hong Kong's status as an international financial center has attracted a lot of capital to undertake investment through the Hong Kong market. With mature institutional support, such as unrestricted capital flows, a simple tax system and a free flow of information, the stock-oriented approach should be the prevailing theory for the Hong Kong stock market performance. In light of the linked exchange rate system, an appreciation of the US dollar (USD) will also bring up the HKD's value. This results in capital inflow into the Hong Kong market yielding a higher return. HKEER can therefore be used to measure capital inflow and outflow.

As for INT, it is used to measure the cost of doing business and the riskiness of investment. It should inversely relate to stock market performance.

The SHA and SPX, on the other hand, are representing the two major stock markets that may have compelling impacts on the Hong Kong stock market. Previous studies have concluded that the Chinese stock market and the U.S. stock market were cointegrated with the Hong Kong stock market (Aloy et al., 2013; Su et al., 2007). Therefore, to a certain extent, Hang Seng sub-indexes should move with them in the same direction. Considering the fact that the U.S. and China are the two largest economies in the globe, and they are major trading partners of Hong Kong, these two variables can be perceived as the proxies for measuring economic performance and outlook outside Hong Kong. They exhibit the effect of non-local aspects on the sub-indexes.

As this paper focuses on sector sub-indexes, industry-specific variables should also be considered in each model, so as to sharpen the illustration of the influence of those variables on the stock price among various sectors in Hong Kong.

C&I traces the stock prices of companies, listed in the Stock Exchange of Hong Kong (SEHK), that belong to the commerce industry sector. As shown in Table 5.1, several industries like services, industrial and consumer goods are involved; therefore, the value of trade (TRADE) is added to the model as an indicator to reflect the performance of these industries, as in Equation (3) shown below.

$$f(\underset{-}{CPI},\ \underset{+}{MS},\ \underset{+}{HKEER},\ \underset{-}{INT},\ \underset{+}{SHA},\ \underset{+}{SPX},\ \underset{+}{\boldsymbol{TRADE}}) \tag{3}$$

FIN measures the SEHK-listed companies that engage in the financial sector of Hong Kong. In this sector, whether the banking sector can make a profit relies on debt financing made by borrowers. The value of new loans drawn (NEWLOAN) in each period can measure the state of this sector. The model for FIN is devised as below.

$$f(\underset{-}{CPI},\ \underset{+}{MS},\ \underset{+}{HKEER},\ \underset{-}{INT},\ \underset{+}{SHA},\ \underset{+}{SPX},\ \underset{+}{\boldsymbol{NEWLOAN}}) \tag{4}$$

PROP mirrors the stock performance of enterprises in the property sector of Hong Kong. As the constituents are property developers, it is believed that property price (PROPP) can reflect those companies' overall performance.

$$f(\underset{-}{CPI},\ \underset{+}{MS},\ \underset{+}{HKEER},\ \underset{-}{INT},\ \underset{+}{SHA},\ \underset{+}{SPX},\ \underset{+}{\boldsymbol{PROPP}}) \tag{5}$$

UTI measures the SEHK-listed companies that provide utility services to the general public, commonly including electricity supply and town gas. Although natural gas and coal prices could reflect the profitability in the sector, the firms in the industry usually transfer the business cost to their customer; but they are, at the same time, constrained by the Scheme of Control Agreement. Hence, the UTI is a special case that does not include any industry-specific variable in the model. Besides, expectations on CPI may be uncertain. Although CPI is negatively correlated to stock price based on past empirical studies, it is believed that there might be a positive relation between them for two reasons. Firstly, the price fluctuations of the utility stocks are less volatile than those of other sectors; in other words, it is a less risky investment from the investors' perspective. Second, the increase in costs is transferred to the consumers through increasing charges. The lack of substitutes in the sector makes it easier for the firms to raise their product or service prices. The positive linkage between their pricing and CPI is much stronger than companies in other industries. With these two characteristics, some investors would consider utilities stock as a tool to hedge against inflation. So, the UTI model is constructed as follows.

$$\underset{+/-}{f(CPI,} \quad \underset{+}{MS,} \quad \underset{+}{HKEER,} \quad \underset{-}{INT,} \quad \underset{+}{SHA,} \quad \underset{+}{SPX)} \tag{6}$$

## 5.4 METHODOLOGY

To study the relationship between stock indexes and economic variables, the Johansen cointegration test is employed after testing the stationarity of the data. If the cointegration is confirmed, a vector error correction model (VECM) will be built. Granger causality, impulse response (IR) and variance decomposition (VDC) are then used to explain the interactions between various sub-indexes and variables.

Unlike vector autoregressive model (VAR), VECM representation incorporates both long-run and short-run effect of the variables. It allows the model to include the error correction terms (ECTs), representing the force or tendency towards the theoretical equilibrium state.

### 5.4.1 VECM and Johansen Cointegration Testing

Johansen (1991) devised the VECM, starting from the transformation of the g-dimensional VAR model (with k lags), given that $t = 1, \ldots, T$, where T is the total number of observations,

$$y_t = \delta_1 y_{t-1} + \delta_2 y_{t-2} + \cdots + \delta_k y_{t-k} + \mu + u_t \tag{7}$$

into the VECM representation with (k-1) lags:

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \cdots + \Gamma_{k-1} \Delta y_{t-(k-1)} + \mu + u_t \tag{8}$$

In Equation (7), the mean $\mu$ and the residuals $u_t$ are g×1 vectors; the vector $y$ and its different lags, including $y_t, y_{t-1}, y_{t-2}, y_{t-k}$ , are also g×1 vectors; $\delta_1$ to $\delta_k$ are g×g matrices. In Equation (8), the long run coefficient $\Pi = \left(\sum_{i=1}^{k} \delta_i\right) - I$ and the coefficient of the first difference of variables $y$, $\Gamma_i = -\left(\sum_{j=i+1}^{k} \delta_j\right)$ are g×g matrices. It should be noted that in both Equations (7) and (8), the residuals are $u_t \sim (0, \Sigma)$.

The application of VECM requires the components to be stationary or I(0); but the fact is that economic data often involve non-stationary variables which are I(1). Those I(1) variables become I(0) after taking the first difference, meaning that all $\Delta y$ terms in Equation (8) are I(0); so, the validity of using VECM lies on the stationarity of the term $\Pi y_{t-1}$.

Johansen's technique tests the rank (r) of the $\Pi$ matrix. This long-run coefficient can be expressed as

$$\Pi = \alpha\beta' = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{\mathrm{r1}} \\ \vdots & \ddots & \vdots \\ \alpha_{\mathrm{1g}} & \cdots & \alpha_{\mathrm{rg}} \end{bmatrix} \begin{bmatrix} \beta_{11} & \cdots & \beta_{\mathrm{1g}} \\ \vdots & \ddots & \vdots \\ \beta_{\mathrm{r1}} & \cdots & \beta_{\mathrm{rg}} \end{bmatrix} \tag{9}$$

where the matrix $\alpha$ contains information about the speed of adjustment and the matrix $\beta'$ is the transpose of a cointegrating matrix. r is defined as the rank of matrix $\Pi$ or the number of cointegrating vectors. In the decomposition of $\Pi$ shown in Equation (9), the elements in $\alpha$ indicate the speed of adjusting the disequilibrium toward the equilibrium state. The coefficients in $\beta$ illustrate the roles of variables in the long-run equilibrium. To interpret the coefficients, $\beta'$ Equation (9) is usually normalized on a particular variable, so that one of the coefficients will be set to be one. Simply put, a normalized ECT in general form is

$$\left(y_{1t-1} - \beta_{12}y_{2t-1} - \cdots - \beta_{\mathrm{1g}}y_{\mathrm{gt-1}}\right) \tag{10}$$

where $\beta_{ij}$, (j= $2, \ldots, \mathrm{g}$), is the normalized long-term parameter for $y_j$ of the $i$-th cointegrating vector.

It is possible to include an intercept term in the cointegrating equation, but it is left out for ease of illustration. For a $\Pi$ matrix in which the number of ranks equals r, there will be r columns of $\beta$ and r ECTs.

Referring back to Equation (9), if the cointegration exists, there is at least one cointegrating vector and $\beta' y_t$ is stationary. In other words, all parts of Equation (8) are stationary, which in turn justifies the use of VECM. The number of cointegrating vectors should be 0 < r < g. If r = g, it implies that variables in the level are stationary. If r = 0, VECM is not applicable.

The number of cointegrating relations is tested using the trace statistics which test the null hypothesis that the number of cointegrating equations equals r against the alternative of g:

$$\mathrm{Trace} = -\mathrm{T}\sum_{\mathrm{i=r+1}}^{\mathrm{g}} \ln(1 - \lambda_{\mathrm{i}}) \tag{11}$$

where $\lambda_i$ is the $i$th ordered eigenvalue from the $\Pi$ matrix. The small-sample adjustment is done by replacing T with (T-g(k-1)) as suggested in Reimers

(1992). Actually, in order to determine the number of ranks, the common practice is to either use the trace test solely or use the trace test and the maximal eigenvalue tests. We apply the trace test only because the power of the trace test is superior (Lutkepohl et al., 2001).

Once the cointegrating relations are found, the VECM can capture the short-run error-correction process through which the variables move to adjust toward equilibrium.

### 5.4.2 Granger Causality

Granger causality analysis is used to examine the predictability of future changes of the variables based on their past values. The VECM (Equation 8) can be rewritten as follows,

$$\Delta y_t = \alpha\beta' y_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta y_{t-j} + \mu + u_t \qquad (12)$$

It is obvious that the VECM consists of mainly two parts, the lagged stationary linear combination arising from the cointegrating relationship and the previous value of the variable itself. The change in dependent variables lies on the level of the lagged disequilibrium in the cointegrating relationship, captured by the ECT ($\beta' y_{t-1}$), and the changes in lagged differences of the variables in the system. These settings allow us to readily distinguish between long-run Granger causality and short-run Granger causality.

When the variables under study are cointegrated, variables in level move together and hence are said to have a long-run equilibrium. Any disequilibrium induced by any shocks is accounted for in the ECT. If ECT increases, the deviation will cause changes in the dependent variable in order to adjust to restore the equilibrium. It is known as the long-run Granger causality. In addition to the long-run Granger causality induced through the ECT, the change in the dependent variables also responds to short-term shocks to the stochastic environment through $\sum_{j=1}^{k-1} \Gamma_j \Delta y_{t-j}$ in Equation (12).

As a result, the long-run and short-run Granger causalities can be inspected by testing the statistical significance of the lagged ECTs and the sum of the lags of each explanatory variable $\sum_{j=1}^{k-1} \Gamma_j \Delta y_{t-j}$ respectively. By t-test, coefficients of the ECTs, the elements in $\alpha$ are tested under the null hypothesis, $H_0: \alpha = 0$ v.s. the alternative hypothesis: $H_1: \alpha \neq 0$. If the null hypothesis $H_0$ is rejected, the ECT is statistically significant. The elements in $\alpha$ can represent the proportion of disequilibrium being corrected in each period. A joint test, either the F or Wald test, is applied to test the (first-differenced) explanatory variables by imposing zero restrictions.

### 5.4.3 Impulse Response (IR) and Variance Decomposition (VDC)

VECM and Granger causality analysis cannot explain how long the dynamic interactions take place whereas impulse response function (IRF) and variance decomposition (VDC) allow us to analyze the dynamic properties of the system. Impulse response measures effect of a shock to a variable on other variables, with the assumption that no further shocks shall be added. Through this analysis, the transmission of the effect between variables within a period can be mapped out. VDC explains the system dynamic by measuring how much of the forecast variation can be explained by innovations in all variables including its own. VDC can help measure the relative strength of the Granger-caused chain beyond the sample period. This paper focuses on the sub-indexes response to any innovation added in other explanatory variables, for example, the effect on C&I's dynamic response path of a shock to CPI. It also applies VDC to see how each sub-index's variance can be explained by innovations in economic variables in the system.

## 5.5 DATA

Data for Hang Seng sub-indexes used in this study include monthly data of the Hang Seng Commerce & Industry (C&I) Sub-index, Hang Seng Finance (FIN) Sub-index, Hang Seng Properties (PROP) Sub-index and Hang Seng Utilities (UTI) Sub-index. These sub-indexes are free float-adjusted market capitalization indexes of companies in specified groups of industries as defined in Table 5.1.

Economic variables consist of CPI, MS, HKEER, INT, SHA, SPX, TRADE, NEWLOAN and PROPP (Section 3). Data of the composite consumer price index and the trade-weighted effective exchange rate index are represented by CPI and HKEER, respectively, as updated by the Census and Statistics Department. The composite CPI includes the overall prices paid by households. The trade-weighted effective exchange rate index measures the movements in the exchange rate of Hong Kong dollar (HKD). When it goes up, it means that the strength of HKD increases, and it appreciates against the foreign currencies in the index. The data for M3, updated by the Hong Kong Monetary Authority (HKMA) each month, is used to measure the MS which includes legal tender notes and coins, savings and time deposits and negotiable certificates of deposit issued by all deposit-taking institutions in the three-tier banking system. Data for INT are collected through DataStream and are represented by the Hong Kong Government Benchmark Bid Yield 10 Years. The reason for choosing the 10-Year yield is that long interest rates can reflect the costs and the long-term risk of business financing more comprehensively while short-term interest rates reflect mainly the short-term liquidity issues.

Shanghai Stock Exchange A Share Index and S&P 500 are represented by SHA and SPX as specified in Section 3. SHA provides a measure of the performance of all A shares listed on the Shanghai Stock Exchange. Despite the restrictions on foreign investors, SHA can imply the current economic condition and the public's confidence toward a future economic environment in Mainland China. Hence, it can represent the economic indicators of Mainland China. SPX traces the performance of 500 companies with the largest market capitalization listed in the New York Stock Exchange (NYSE) or Nasdaq. It covers about 80% of the total market capitalization in the American stock market and represents the general condition of the US economy. Historical values of SHA and SPX are retrieved from Bloomberg data.

TRADE, NEWLOAN and PROPP are measures of external merchandize trade, new loans drawn and the Centa-City Index, respectively, gathered from the Census and Statistics Department, the HKMA and the Centadata. The external merchandise trade includes all imports, domestic exports and re-exports of goods in HKD. The new loans drawn are announced by the HKMA every month. This is a residential mortgage survey. The Centa-City Index is computed based on transactions registered with the Land Registry in a particular month. Thus, the data can embody the corresponding industry-specific factors that the models require to be examined.

In order to study the Hong Kong stock market over the past 10 years, the monthly data cover the period from January 2004 to August 2014, with 128 observations in total. All data are taken from the natural logarithm and seasonally adjusted by the X12 procedure.

## 5.6 EMPIRICAL RESULTS

### 5.6.1 Unit Root Test

The first step of the cointegration analysis is to test for the existence of a unit root in the variables under study. The Augmented Dickey-Fuller (ADF) test, with an intercept and a linear trend in the test equation, is used. Table 5.3 shows we cannot reject the null of a unit root for all variables in level, but we can reject the null against the alternative of stationarity for all in the first difference. Hence, we conclude that the variables in level are integrated into order 1 or I(1).

**Table 5.3. Unit root test**

| Variable In Level | ADF | Variable in the first Difference | ADF |
|---|---|---|---|
| C&I | -1.906 | ΔC&I | -4.440*** |
| FIN | -2.487 | ΔFIN | -4.810*** |
| PROP | -2.692 | ΔPROP | -4.840*** |
| UTI | -2.864 | ΔUTI | -4.962*** |
| CPI | -1.584 | ΔCPI | -4.790*** |
| MS | -1.926 | ΔMS | -12.37*** |
| HKEER | -1.866 | ΔHKEER | -3.825** |
| INT | -2.275 | ΔINT | -6.744*** |
| SHA | -2.632 | ΔSHA | -3.390* |
| SPX | -1.200 | ΔSPX | -3.916** |
| TRADE | -2.556 | ΔTRADE | -19.14*** |
| NEWLOAN | -2.545 | ΔNEWLOAN | -4.318*** |
| PROPP | -2.328 | ΔPROPP | -3.751** |

_Notes: An intercept and a linear trend are included in the test equation. The number of lag length chosen in the test equation is based on the modified Akaike information criteria. *, ** and *** denote statistical significance at the 10%, 5% and 1% level respectively._

### 5.6.2 Johansen Cointegration Test

Next, we apply the Johansen cointegration test for analysis. We focus on what economic variables are required in cointegrating vectors to affect the sub-indexes. The cointegration test is then accompanied by the exclusion test on the long-run coefficients so that the variables that are statistically insignificant in the cointegrating vector in each sub-index model are excluded. The results of cointegration conducted without insignificant economic variables in the VECM and the variables included in the cointegrating vector in each Hang Seng sub-index model are shown in Table 5.4. The results of the Trace statistics after the small-sample adjustments indicate that there is at the most one cointegrating relationship among the variables under study.

All the long-run coefficients in the cointegrating vector $\beta = (1, -\beta_2, \dots, \beta_i)'$ are statistically different from zero in the VECMs. This result confirms that each sub-index is moving together with macroeconomic and industry-specific variables; some of them also relate to the China economy in the long run. If we compare the cointegrating vectors between models, it is obvious that the roles of economic variables in each sub-index variation are quite different in the long run. There are a few points we would like to highlight here.

Firstly, the inclusion of SPX does not mean rejection of the cointegration in each sub-index model, but surprisingly it is not statistically significant. It means that the American stock market is not required to maintain a long-term relationship with the Hong Kong stock market. If we consider SPX as a proxy for the U.S. economy as we initially suggested in Section 3, it implies that the long-run influence from the U.S. economy on the Hong Kong stock market is too weak to be included in the cointegrating vector. Instead, SHA is significant in the sub-indexes of the C&I, FIN and PROP models. The result of the co-movement of stock markets in China and Hong Kong is consistent with that of Su et al. (2007). In fact, Chinese enterprises are taking larger shares in the HSI as shown in Table 5.2. It is estimated that 45.13% of the total market capitalization in HSI comes from current constituents of HSCEI and HSCCI in December 2014, as shown in Appendix 1.

Secondly, CPI is found to be significant in the cointegrating relationship in all models but FIN. This result suggests that financial institutions might be able to transfer the inflation cost to their borrowers as their source of revenue is the interest differential between investment and funding. In this case, Equation (1) implies that the value of the stock is unaffected by the inflation effect in $\mathrm{CF}$, and $(r_f + r_p)$ offset one another. Another theory that might be a potential explanation is that the performances of international banks and Chinese banks are not statistically significantly affected by the local CPI. Based on either explanation, the FIN model rejects the proxy theory suggested by Fama (1981).

Thirdly, unlike other models, MS and SHA are rejected in the cointegrating vector of the UTI model. This is in consonance with the stable growth of the trend witnessed from 2004 to 2014 (Fig. 5.1). Utility stocks have been perceived as a “safe haven” in times of financial crises. In the period between September 2007 and December 2008, the UTI declined about 7% while other sub-indexes had fallen about 50%. It appears that the UTI is immune to financial distress to a certain degree. As mentioned in Section 3, MS could affect stock price through liquidity effect, earnings effect and risk premium (Hamburger & Kochin 1972). This does not apply to the UTI model. Change in MS would not change the market capitalization in the utilities sector because the Scheme of Control Agreement effectively regulates the earnings of those companies through permitted return and tariff review. With the immune nature of UTI, the insignificance of SHA in the cointegrating vector is fairly plausible.

After that, we move on to examine the signs of long-run coefficients in the cointegrating vector in each model. In general, our results meet the expectations stated in Section 3.

CPI negatively relates to the C&I and PROP models and that affirms Fama’s (1981) theory of the proxy effect of inflation which has a negative influence over the real economy (Cohn & Lessard, 1981; Fama, 1981). In theory, an increase in the price level shrinks the real money supply, driving up the cost of money and frustrating the stock price as well as real economic activities. However, the UTI model is a special case in which an increase in CPI has a positive influence on the UTI. The implication of this outcome is that the UTI is used as a hedge against poor economic conditions and inflation. A jump in CPI is considered bad news for the C&I and PROP because it leads to a decline in the real money supply and hence real economic activities. As a result, investors might be inclined to choose a safer investment alternative like stocks in the utilities sector. Furthermore, cost-push inflation could drive up prices like electricity charges; some investors invest in utility stocks in the UTI because they believe these companies’ earnings are not affected much by this type of inflation; their ability to pass on the increase in cost to buyers makes it to be perceived as an inflation-protected investment, in spite of the Scheme of Control Agreement.

MS in Hong Kong shows a positive effect on the C&I, FIN and PROP. When MS increases, the stock price is indeed affected by the overwhelming effects from income (Hamburger & Kochin, 1972). The MS benefits the real income of individual. As the money demand theory suggests, an increase in MS increases the cash flow in companies (Equation (1)). As for the liquidity effect, an easy money state in the economy drives companies’ funding costs down. However, it is believed that this liquidity effect is weakened since the linked exchange rate system limits the short-term interest rate movement. So, how large the liquidity effect depends more on the shift of medium-term and long-term interest rates in response to the MS increase.

As shown in the cointegrating vector in each model, the exchange rate and long-term interest rate in Hong Kong are important driving forces in the long run. The negative coefficient of the HKEER supports the stock-approach theory in that the value of HKD increases with stock prices in Hong Kong. A rise in HKEER represents the appreciation of HKD and thus implies capital inflows into Hong Kong. As part of the capital goes to the equity market and some of them contribute to the real economy, the stock price benefits from larger demand for stocks. INT, which can be regarded as an indicator for country risk and cost of capital, exhibits an opposite tie to the stock prices in the results. Any gloomy outlook of the economy can lift the interest rates and lead to a fall in the sub-indexes.

As far as the industry-specific variable is concerned, TRADE, NEWLOAN and PROPP have a significantly positive relationship with C&I, FIN and PROP, respectively in the long run. This is consistent with our expectations shown in Section 3. First, the larger the trade value, the more profitable are companies in the commerce and industry sector, meaning higher values of stocks, as illustrated by the C&I model. But in the FIN model, NEWLOAN is marginally accepted as a part of cointegration at the 0.1 level. Without a doubt, an increase in new loans drawn means higher profitability; one should be aware that NEWLOAN is a local economic variable while constituents of FIN are mostly international banks or Chinese banks. This should have been explained by the NEWLOAN variable in the FIN model to a lesser extent. In the PROP model, although most of the companies have engaged in the Hong Kong property market, the coefficient of PROPP is still relatively small despite its significance because of the government intervention in the property market and diversification of business portfolios by the local property companies, like investing in Chinese or foreign property market and non-property sectors. For these reasons, the linkage of the property price and stock price is diluted.

If we compare the long-run parameters $\beta$ among models, we can easily notice the important role HKEER and INT play in the cointegrating relationship. So the long-run equilibriums of the sub-indexes are substantially determined by these two variables, implying the capital flow and local long-term interest rate are major determinants of sub-index movements. According to the results, 1% increase in HKEER can increase C&I, FIN, PROP and UTI by 6.39%, 6.34%, 7.07% and 3.99% respectively, whereas 1% increase in INT causes 8.5%, 8.2%, 17% and 18% loss in the C&I, FIN, PROP and UTI respectively. The large coefficients of INT in PROP and UTI models shed some light on how the sub-indexes reflect the nature of the industries. For the property sector, an increase in interest rate depresses the property market outlook and worsens the prospect of future cash flow and it also increases the cost of doing business. As regards the utilities sector, the INT, which echoes the cost and risk of businesses in other sectors, is extremely sensitive to UTI which provides an alternative choice for less risky investments.

**Table 5.4. Johansen cointegrating testing and the long-run parameters**

| **Panel A: Hang Seng Commerce and Industry Sub-index (C&I)** | | | | | |
|---|---|---|---|---|---|
| $\mathbf{H_0}$ | $\boldsymbol{\lambda_r}$ | **Trace** | **Adjusted Trace** | **10% C.V.** | **5% C.V.** |
| $r = 0$ | 0.3982 | 152.30 | 135.64*** | 120.37 | 125.62 |
| $r \leq 1$ | 0.2279 | 87.290 | 77.743 | 91.110 | 95.754 |
| $r \leq 2$ | 0.1653 | 54.178 | 48.252 | 65.820 | 69.819 |
| $r \leq 3$ | 0.1171 | 31.046 | 27.650 | 44.494 | 47.856 |
| $r \leq 4$ | 0.0764 | 87.290 | 13.447 | 27.067 | 29.797 |
| $r \leq 5$ | 0.0296 | 54.178 | 4.3833 | 13.429 | 15.495 |
| $r \leq 6$ | 0.0083 | 31.046 | 0.9522 | 2.7055 | 3.8415 |
| *Coefficients of Johansen Cointegrating Vector (C&I is normalized)* | | | | | |
| **CPI** | **MS** | **HKEER** | **INT** | **SHA** | **TRADE** |
| 5.6945*** | -2.5244*** | -6.3914*** | 8.4999*** | -0.2145*** | -1.3830*** |
| (1.2505) | (0.4525) | (1.0735) | (3.2477) | (0.0804) | (0.2582) |

| **Panel B: Hang Seng Finance Sub-index (FIN)** | | | | | |
|---|---|---|---|---|---|
| $\boldsymbol{H_0}$ | $\boldsymbol{\lambda_r}$ | **Trace** | **Adjusted Trace** | **10% C.V.** | **5% C.V.** |
| $r = 0$ | 0.3560 | 125.27 | 113.53*** | 91.110 | 95.754 |
| $r \leq 1$ | 0.2079 | 68.940 | 62.477 | 65.820 | 69.819 |
| $r \leq 2$ | 0.1673 | 39.115 | 35.448 | 44.494 | 47.856 |
| $r \leq 3$ | 0.0753 | 15.674 | 14.205 | 27.067 | 29.797 |
| $r \leq 4$ | 0.0279 | 5.6489 | 5.1193 | 13.429 | 15.495 |
| $r \leq 5$ | 0.0157 | 2.0308 | 1.8404 | 2.7055 | 3.8415 |

*Coefficients of Johansen Cointegrating Vector (FIN is normalized)*

| **MS** | **HKEER** | **INT** | **SHA** | **NEWLOAN** |
|---|---|---|---|---|
| -1.1278*** | -6.3405*** | 8.1970** | -0.2147*** | -0.1066* |
| (0.2317) | (1.2438) | (3.5489) | (0.0639) | (0.0640) |

| **Panel C: Hang Seng Properties Sub-index (PROP)** | | | | | |
|---|---|---|---|---|---|
| $H_0$ | $\lambda_r$ | **Trace** | **Adjusted Trace** | **10% C.V.** | **5% C.V.** |
| $r = 0$ | 0.3609 | 152.37 | 127.37** | 120.37 | 125.62 |
| $r \leq 1$ | 0.2203 | 95.077 | 79.478 | 91.110 | 95.754 |
| $r \leq 2$ | 0.1957 | 63.223 | 52.851 | 65.820 | 69.819 |
| $r \leq 3$ | 0.1231 | 35.354 | 29.553 | 44.494 | 47.856 |
| $r \leq 4$ | 0.0844 | 18.533 | 15.492 | 27.067 | 29.797 |
| $r \leq 5$ | 0.0431 | 7.2421 | 6.0540 | 13.429 | 15.495 |
| $r \leq 6$ | 0.0125 | 1.6074 | 1.3437 | 2.7055 | 3.8415 |
| *Coefficients of Johansen Cointegrating Vector (PROP is normalized)* | | | | | |
| **CPI** | **MS** | **HKEER** | **INT** | **SHA** | **PROP** |
| 2.6956* | -1.3485*** | -7.0698*** | 17.070*** | -0.3489*** | -0.7460*** |
| (1.3621) | (0.5132) | (1.2785) | (3.7603) | (0.1016) | (0.2555) |
| **Panel D: Hang Seng Utilities Sub-index (UTI)** | | | | | |
| $H_0$ | $\lambda_r$ | **Trace** | **Adjusted Trace** | **10% C.V.** | **5% C.V.** |
| $r = 0$ | 0.2623 | 67.074 | 62.849*** | 44.494 | 47.856 |
| $r \leq 1$ | 0.1334 | 28.438 | 26.646 | 27.067 | 29.797 |
| $r \leq 2$ | 0.0765 | 10.251 | 9.6054 | 13.429 | 15.495 |
| $r \leq 3$ | 0.0012 | 0.1461 | 0.1369 | 2.7055 | 3.8415 |
| *Coefficients of Johansen Cointegrating Vector (UTI is normalized)* | | | | | |
| **CPI** | | **HKEER** | | **INT** | |
| -2.4307*** | | -3.9857*** | | 18.183*** | |
| (0.7384) | | (1.0476) | | (4.5564) | |

*Notes: Assume that the level data have linear deterministic trends and there are intercepts but no trend in the cointegrating equations*
*Long-run coefficients in cointegrating vector is shown as β = (1, -$\beta_2$, … , -$\beta_i$)' such that the first variable, sub-index, is normalized to be unity. The number of lag length chosen in the test equation is based on the Akaike information criteria. Adjusted trace statistics were adjusted from trace statistics by Reimer's method of small-sample modification (Reimer 1992). $\lambda_r$ denotes eigenvalues. 10% C.V. and 5% C.V. stand for 0.10 critical value and 0.05 critical value respectively*
*Standard error is in (). *, ** and *** denote statistical significance at the 10%, 5% and 1% level respectively.*

**Table 5.5. Summary for Long-run and Short-run Granger Causality**

| | | | | | |
|---|---|---|---|---|---|
| **Long-run Granger Causality** | | | | | |
| **Panel A: Hang Seng Commerce and Industry Sub-index (C&I)** | | | | | |
| CPI→C&I | MS→C&I | HKEER→C&I | INT→C&I | SHA→C&I | TRADE→C&I |
| | | C&I→HKEER | | C&I→SHA | C&I→TRADE |
| **Panel B: Hang Seng Finance Sub-index (FIN)** | | | | | |
| | MS→FIN | HKEER→FIN | INT→FIN | SHA→FIN | NEWLOAN→FIN |
| | | | FIN→INT | FIN→SHA | FIN→NEWLOAN |
| **Panel C: Hang Seng Properties Sub-index (PROP)** | | | | | |
| CPI→PROP | MS→PROP | HKEER→PROP | INT→PROP | SHA→PROP | PROPP→PROP |
| | | | PROP→INT | | PROP→PROPP |
| **Panel D: Hang Seng Utilities Sub-index (UTI)** | | | | | |
| CPI→UTI | | HKEER→UTI | INT→UTI | | |
| UTI→CPI | | | UTI→INT | | |
| **Short-run Granger Causality** | | | | | |
| **Panel A: Hang Seng Commerce and Industry Sub-index (C&I)** | | | | | |
| CPI→C&I | | | INT→C&I | | |
| | C&I→MS | C&I→HKEER | | | C&I→TRADE |
| **Panel B: Hang Seng Finance Sub-index (FIN)** | | | | | |
| | | HKEER→FIN | INT→FIN | | |
| | | FIN→HKEER | FIN→INT | | |
| **Panel C: Hang Seng Properties Sub-index (PROP)** | | | | | |
| | PROP→MS | PROP→HKEER | | | PROP→PROPP |
| **Panel D: Hang Seng Utilities Sub-index (UTI)** | | | | | |
| | | HKEER→UTI | | | |
| | | UTI→HKEER | | | |

## 5.6.3 Long-run Granger Causality

For examining causal relationships in the long run, we carry out Granger causality tests in this section. Using t-test, the first element in $\alpha$, which is the coefficients of the ECT, is statistically significant for all equations of sub-indexes, suggesting that lagged disequilibrium is corrected toward long-run equilibrium in every period. In other words, economic variables in each cointegrating vector Granger-cause each corresponding sub-index. When taking C&I equation in Panel A of Table 5.6 as an example, CPI, MS, HKEER, INT, SHA and TRADE Granger-cause C&I. At the same time, C&I is Granger-caused by HKEER, SHA and TRADE in the long run and therefore they have bi-directional long-run Granger causality with C&I. Table 5.5 shows the summary for directions of the long-run Granger causality. In addition, bi-directional causality is found between the sub-index and its corresponding industry-specific variable in each model. This phenomenon implies that the favorable development of the industry boosts the stock price in that sector and a higher stock price leads to better business performance of companies, probably because the higher stock price creates a wealth effect and credit price effect (Chen, 2001) benefiting the industry itself and it also results in a better economic environment which encourages credit activities.

As the main focus lies on how the sub-index responds to the disequilibrium, based on values of $\alpha$ in Table 5.6, 19.5%, 19.96% and 27.73% of the disequilibrium is eliminated by the changes in C&I, FIN and PROP, respectively. Also, only 5.13% of deviation from long-run equilibrium is corrected by UTI. The slow adjustments in the UTI model support the observation that utilities stocks in the UTI do not actively trade and the UTI does not react much to fluctuation in some macro-economic variables in each period. The UTI adjusts slowly so that it fully turns the system back to the long-run equilibrium level over a long period of time.

## 5.6.4 Short-run Granger Causality

For the short-run Granger causality, we test it by using the standard Wald statistics (Table 5.6), which find that the bi-directional Granger causality is no longer significant between the sub-index and industry-specific variables in each model. However, it shows that movements of the C&I and PROP still Granger-cause the TRADE and PROPP respectively in the short run. It reveals that the increase in the stock price in C&I brings wealth effect and promotes trading activities in both the long run and short run. The causality of PROPP from the PROP might suggest that buyers and sellers of properties take the profitability of real estate developers and sales of new houses as references in their pricing decisions in the short as well as long run. The higher (lagged) property price level in turn allows local developers to profit from it, as they set the prices by referring to the property market in Hong Kong.

SHA is found to have no short-run Granger causality in any direction with sub-index, implying that SHA movement does not affect Hang Seng sub-indexes in the short run. Besides, the variation in stock price could create fluctuations in HKEER and real activities in some sectors. Those causalities are the outcome of short-run speculation activities going on in the stock market. When stock prices go up, investors respond to the market immediately, for example by selling their holdings for book profit. Some investors might change their consumption patterns as their wealth changes. The summary of direction of the short-run Granger causality is shown in Table 5.5.

### 5.6.5 Impulse Response and Variance Decomposition

While Granger Causality does not show how the sub-indexes respond to changes in explanatory variables dynamically, we use IRF and VDC to assess this dynamic pattern based on the VECM. To avoid the VAR ordering problem, a generalized impulse response is applied (Pesaran & Shin, 1998). The results are generated in Fig. 5.2.

We can see that a positive shock of CPI induces an upward response in C&I and PROP. However, five periods later, the effect of CPI in C&I becomes negative and persistent over time. A similar trend can be observed in PROP. So it takes five to seven periods for the proxy effect to be dominating. The models also show that the responses to money supply are dying down; that is obvious, especially in the FIN and PROP. After 10 periods, the effects of the shocks diminish to near-zero levels. In C&I, FIN and PROP, a shock in HKEER first brings down the sub-index and then grows upward to the positive side within half a year. The initial downward trend is unlikely to be the result of deliberate acts by the investors to manipulate the stock price to a lower level before they invest more money into it, as it takes three or more months to show that positive effect on the stock price. Instead, it might be related to a higher value of HKD which discourages exports of Hong Kong. INT and SHA exhibit a negative impact and a positive impact, respectively, in the stock market. Overall, all models are dynamically stable.

To gauge the source of forecast variation of the sub-index and the relative strength of Granger-causal chains, VDC is estimated (Table 5.7). Most of the variations in the four sub-indexes can be highly explained by their own innovations; especially for UTI; after 12 periods, 84% of its own variance can be explained by its own innovations. This matches with our previous result on UTI in the sense that the disequilibrium is adjusted through UTI with the smallest value of $\alpha$ in absolute terms, indicating that UTI tends to be more exogenous than other sub-indexes. All other three models consistently show that the proportions of variance explained by them are fading off gradually over time and it implies that their variances are explained by other variables. For example, in the FIN model, 35.6% of the variance can be explained by innovations in HKEER. HKEER is clearly a significant contributor to the forecast error of C&I and PROP.

NEWLOAN in the FIN model and PROPP in the PROP model are found to be less important in explaining the forecast error variance of their corresponding sub-indexes. Comparing these four sub-indexes, INT and SHA are particularly important as their shocks explain 12.751% and 21.611% of the variance of PROP. Moreover, CPI is relatively more important in C&I than in other models since commercial and industrial businesses are more influenced by production costs than other variables. Innovations in CPI explain nearly 20% of the variance of C&I.

## Table 5.6. Long-run and Short-run Granger Causality Tests

**Panel A: Hang Seng Commerce and Industry Sub-index (C&I)**

| **Dep Var** | **Block Exogeneity Wald Tests ($\chi^2$)** | | | | | | | **Error Correction Term** | |
|---|---|---|---|---|---|---|---|---|---|
| | **ΔC&I** | **ΔCPI** | **ΔMS** | **ΔHKEER** | **ΔINT** | **ΔSHA** | **ΔTRADE** | **$\alpha$** | **t statistic** |
| ΔC&I | | 7.235** | 1.530 | 2.323 | 4.723* | 4.452 | 2.526 | -0.1975 | [-4.535]*** |
| ΔCPI | 0.608 | | 4.878* | 6.678** | 1.826 | 3.022 | 1.645 | 0.0021 | [0.506] |
| ΔMS | 11.23*** | 0.906 | | 3.661 | 0.451 | 4.704* | 3.718 | -0.0035 | [-0.293] |
| ΔHKEER | 20.75*** | 7.673** | 4.954* | | 1.580 | 3.198 | 8.461** | 0.0093 | [2.310]** |
| ΔINT | 1.736 | 0.145 | 2.560 | 0.204 | | 2.798 | 3.765 | -0.0015 | [-0.731] |
| ΔSHA | 3.081 | 0.993 | 0.174 | 1.530 | 3.180 | | 0.723 | -0.2012 | [-3.466]*** |
| ΔTRADE | 10.28*** | 9.205** | 2.550 | 0.427 | 2.155 | 0.118 | | 0.1233 | [3.085]*** |

**Panel B: Hang Seng Finance Sub-index (FIN)**

| **Dep Var** | **Block Exogeneity Wald Tests ($\chi^2$)** | | | | | | **Error Correction Term** | |
|---|---|---|---|---|---|---|---|---|
| | **ΔFIN** | **ΔMS** | **ΔHKEER** | **ΔINT** | **ΔSHA** | **ΔNEWLOAN** | **$\alpha$** | **t statistic** |
| ΔFIN | | 1.512 | 10.15*** | 5.989* | 2.118 | 1.556 | -0.1996 | [-5.243]*** |
| ΔMS | 1.958 | | 2.769 | 0.407 | 7.609** | 1.473 | 0.0020 | [0.180] |
| ΔHKEER | 10.99*** | 1.310 | | 1.424 | 0.376 | 1.043 | 0.0043 | [1.075] |
| ΔINT | 6.759** | 4.549 | 1.974 | | 4.524 | 13.63*** | -0.0036 | [-2.150]** |
| ΔSHA | 0.051 | 0.591 | 4.393 | 3.464 | | 0.067 | -0.1553 | [-2.949]*** |
| ΔNEWLOAN | 4.584 | 5.081* | 5.340* | 2.720 | 2.134 | | -0.2359 | [-2.407]** |

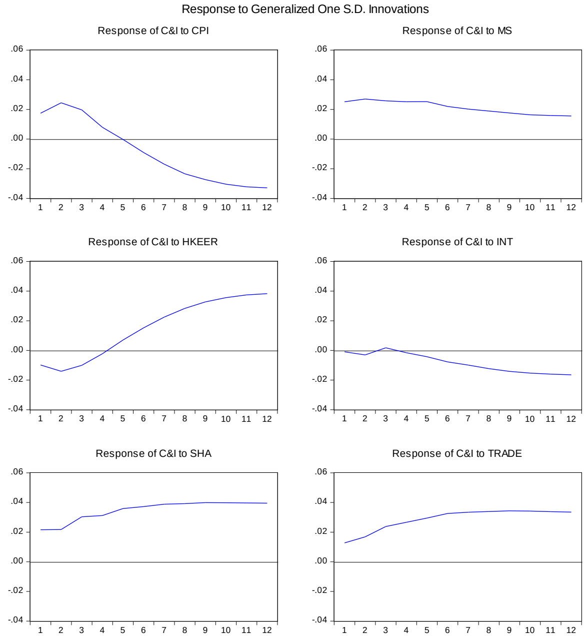
**Panel C: Hang Seng Properties Sub-index (PROP)**

| **Dep Var** | **Block Exogeneity Wald Tests ($\chi^2$)** | | | | | | | **Error Correction Term** | |
|---|---|---|---|---|---|---|---|---|---|
| | **ΔPROP** | **ΔCPI** | **ΔMS** | **ΔHKEER** | **ΔINT** | **ΔSHA** | **ΔPROPP** | **$\alpha$** | **t statistic** |
| ΔPROP | | 6.136 | 1.752 | 4.880 | 4.863 | 2.673 | 3.520 | -0.2773 | [-4.568]*** |
| ΔCPI | 3.458 | | 5.894 | 5.426 | 1.683 | 1.433 | 0.374 | 0.0061 | [1.289] |
| ΔMS | 8.545** | 1.554 | | 1.688 | 1.032 | 11.75*** | 4.566 | 0.0140 | [1.033] |
| ΔHKEER | 13.89*** | 13.67*** | 3.694 | | 4.071 | 5.541 | 3.805 | -0.0018 | [-0.379] |

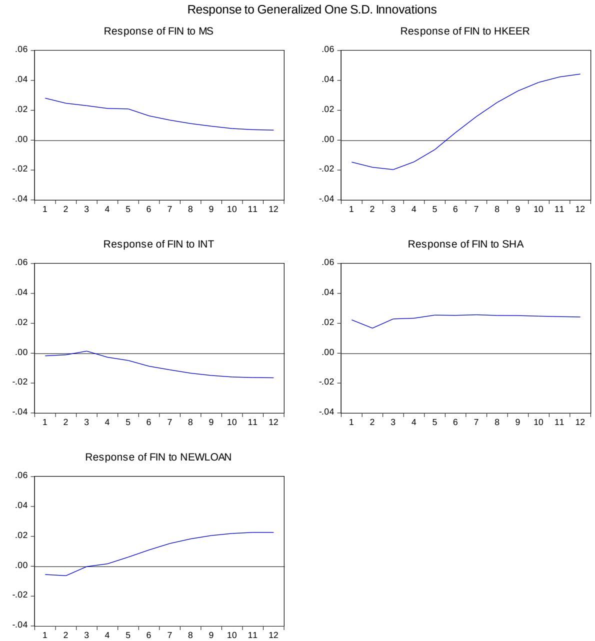
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ΔINT | 3.399 | 5.201 | 8.968** | 6.254* | | 2.494 | 6.716* | -0.0052 | [-2.409]** |
| ΔSHA | 2.761 | 2.383 | 0.041 | 6.791* | 3.428 | | 10.28** | -0.0882 | [-1.343] |
| ΔPROPP | 43.58*** | 3.753 | 1.939 | 3.335 | 0.997 | 0.357 | | -0.0212 | [-1.868]* |

| **Panel D: Hang Seng Utilities Sub-index (UTI)** | | | | | |
|---|---|---|---|---|---|
| **Dep Var** | **Block Exogeneity Wald Tests ($\chi^2$)** | | | | **Error Correction Term** |
| | **ΔUTI** | **ΔCPI** | **ΔHKEER** | **ΔINT** | **$\alpha$** | **t statistic** |
| ΔUTI | | 3.554 | 10.50*** | 2.227 | -0.0513 | [-2.909]*** |
| ΔCPI | 3.211 | | 5.817* | 0.232 | 0.0091 | [2.985]*** |
| ΔHKEER | 17.04*** | 8.843** | | 0.606 | -0.0010 | [-0.322] |
| ΔINT | 1.567 | 0.195 | 1.814 | | -0.0037 | [-2.471]** |

*Notes: Dep Var denotes the dependent variable. *, ** and *** denote statistical significance at the 10%, 5% and 1% level respectively.*

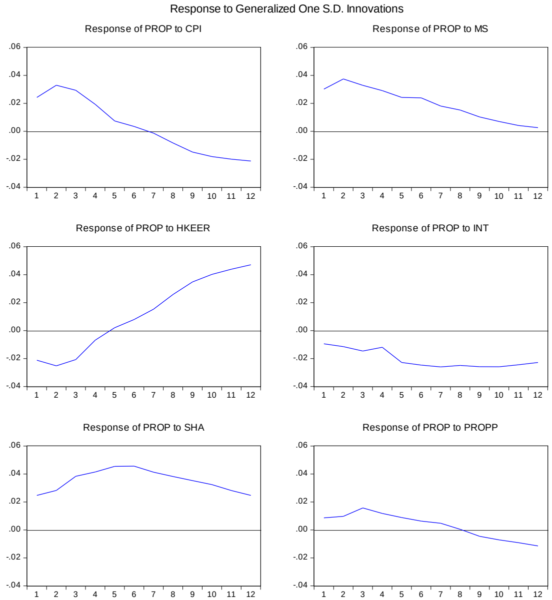



**Hang Seng Commerce and Industry Sub-index (C&I)**

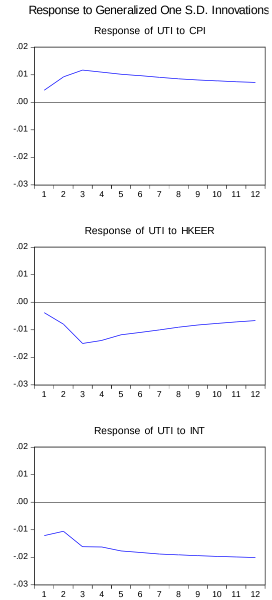



**Hang Seng Finance Sub-index (FIN)**

**Fig. 5.2. Analysis of Impulse Response**




Hang Seng Properties Sub-index (PROP)




Hang Seng Utilities Sub-index (UTI)

**Fig. 5.2. Analysis of Impulse Response (Cont'd)**

**Table 5.7. Variance decomposition analysis**

| **Panel A: Hang Seng Commerce and Industry Sub-index (C&I)** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Period** | **C&I** | **CPI** | **MS** | **HKEER** | **INT** | **SHA** | **TRADE** |
| 1 | 100.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 91.445 | 1.016 | 0.2444 | 0.388 | 0.102 | 2.425 | 4.339 |
| 8 | 60.631 | 10.540 | 0.864 | 8.365 | 1.342 | 8.335 | 9.894 |
| 12 | 38.989 | 19.008 | 1.208 | 15.661 | 2.984 | 10.875 | 11.275 |

| **Panel B: Hang Seng Finance Sub-index (FIN)** | | | | | | |
|---|---|---|---|---|---|---|
| **Period** | **FIN** | **MS** | **HKEER** | **INT** | **SHA** | **NEWLOAN** |
| 1 | 100.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 96.084 | 0.171 | 1.502 | 0.162 | 1.632 | 0.449 |
| 8 | 67.937 | 1.969 | 12.644 | 3.782 | 9.730 | 3.938 |
| 12 | 35.387 | 2.733 | 35.566 | 7.668 | 12.555 | 6.091 |

| **Panel C: Hang Seng Properties Sub-index (PROP)** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Period** | **PROP** | **CPI** | **MS** | **HKEER** | **INT** | **SHA** | **PROP** |
| 1 | 100.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 81.055 | 3.279 | 3.554 | 0.884 | 1.900 | 7.981 | 1.346 |
| 8 | 51.374 | 3.049 | 3.806 | 9.530 | 9.427 | 20.838 | 1.976 |
| 12 | 31.664 | 5.329 | 3.151 | 24.062 | 12.751 | 21.611 | 1.432 |

| **Panel D: Hang Seng Utilities Sub-index (UTI)** | | | | |
|---|---|---|---|---|
| **Period** | **UTI** | **CPI** | **HKEER** | **INT** |
| 1 | 100.000 | 0.000 | 0.000 | 0.000 |
| 4 | 87.769 | 3.934 | 5.965 | 2.332 |
| 8 | 85.542 | 3.597 | 5.168 | 5.693 |
| 12 | 84.266 | 3.074 | 4.068 | 8.591 |

*Notes: Cholesky Ordering for C&I: C&I, CPI, MS, HKEER, INT, SHA, TRADE; Cholesky Ordering for FIN: FIN, MS, HKEER, INT, SHA, NEWLOAN; Cholesky Ordering for PROP: PROP, CPI, MS, HKEER, INT, SHA, PROPP; Cholesky Ordering for UTI: UTI, CPI, HKEER, INT*

## 5.7 CONCLUSION

In this paper, we examine the relationship between Hang Seng sub-indexes and economic variables and compare the response of each sub-index to the macroeconomic and industry environment. The Johansen cointegration test suggests that one cointegrating vector is found in each sub-index model. The economic variables required in the cointegrating vector are not the same in each model. Our estimates indicate that HKEER and INT play indispensable roles in the cointegrating relationship. Also, the long-run coefficients vary in signs and sizes in different models such as INT and CPI. Industry-specific variables like TRADE, NEWLOAN and PROPP appear to have lesser effects on the Hang Seng Sub-indexes. In addition, SPX is statistically insignificant and excluded from all four models, implying that SPX is not as influential as the SHA in the sampling period.

In the VECM, Fama's (1981) proxy effect is found to be a possible explanation for the negative relationship of CPI with the sub-index in the C&I and PROP model. However, the theory is void in FIN and UTI because of the transfer of inflation cost and government control. The effects of MS on sub-indexes show that the earning effect is prominent; when MS rises, C&I, FIN and PROP increase. The role of the HKEER supports the stock-approach theory; capital inflows boost all four sub-indexes in Hong Kong. A negative correlation between INT and sub-indexes is found. Besides, SHA is positively related to C&I, FIN and PROP.

Granger causality test shows that in each period, sub-indexes correct the lagged disequilibrium through ECT. Bi-directional long-run Granger causality is found between sub-indexes and their corresponding industry-specific variables in all models except UTI. This implies that any spur in the industry leads to a spiraling upturn in both the corresponding Hang Seng sub-indexes and business performance in the related industry. Among the four sub-indexes, UTI adjusts its disequilibrium at an exceptionally slow speed, which indicates the less risky nature of its constituents. The overall short-run Granger-causality test results, on the other hand, indicate that stock indexes are affected by short-run movements of economic variables and economic variables are affected by stock indexes in a similar manner.

Impulse response and variance decomposition analysis reaffirm most of the findings in the VECM. It reveals that different sub-indexes are affected by shocks to economic variables in different relative weights.

Overall, this research sheds light on the driving forces of sub-indexes by different economic variables. It explores the differences in responses of different sub-indexes to different macroeconomic variables and industry-specific factors. The implications of the results could help investors to understand the dynamic relationship of the equity market and the fundamentals of the economy, which is critical in investment decision-making.

The limitation of our analysis is the adoption of linear methods without investigating the potential nonlinear relationships studied by for example, Alqaralleh et al. (2021), de Melo & Gomes (2021) and Naifar and Al Dohaiman (2013). We leave the nonlinear analysis to our future research.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

## REFERENCES

Aggarwal, R. (1981). Exchange Rates and Stock Prices: A Study of the US Capital Markets under Floating Exchange Rates. Akron Business and Economic Review, 12(3), 7-12.

Alagidede, P., Panagiotidis, T., & Zhang X. (2011). Causal relationship between stock prices and exchanges rates, Journal of International Trade & Economic Development: An International and Comparative Review, 20 (1), 67-86.

Alqaralleh, H., Al-Majali, A., & Alsarayrh, A. (2021). Analyzing the Dynamics Between Macroeconomic Variables and the Stock Indexes of Emerging Markets, Using Non-linear Methods. International Journal of Financial Research, 12(3), 193-204.

Aloy, M., Boutahar, M., Gente, K. & Peguin-Feissolle, A. (2013). Long-run relationships between international stock prices: further evidence from fractional cointegration tests, Applied Economics, 45(7), 817-828.

Blanchard, O.J. (1981). Output, the stock market, and interest rates, American Economic Review 71(1), 132-143.

Branson, W.H. (1981). Macroeconomic determinants of real exchange rates, National Bureau of Economic Research Working Paper No. 801.

Chen, N.K. (2001). Asset price fluctuations in Taiwan: Evidence form stock and real estate prices 1973 to 1992. Journal of Asian Economics, 12(2), 215-232.

Cohn, R.A. & Lessard, D.R. (1981). The effect of inflation on stock prices: International evidence, Journal of Finance, 36(2), 277-289.

de Melo, M. M., & Gomes, J. W. F. (2021). Long Term Effects of Major Macroeconomic Variables on the Brazilian Stock Market: A nonlinear ARDL application. Archives of Business Research, 9(2).

Dornbusch, R. & Fischer, S. (1980). Exchange rates and the current account, American Economic Review, 70 (5), 960-971.

Fama, E.F. (1981). Stock returns, real activity, inflation, and money, American Economics Review, 71 (4), 545-565.

Gondhalekar V. & Mehdian S. (2003). The-Blue-Monday hypothesis: Evidence based on Nasdaq stocks, 197-2000, Quarterly Journal of Business and Economics, 42 (3/4), 73-89.

Gjerde, O. & Saettem, F. (1999). Casual relations among stock returns and macroeconomic variables in a small, open economy, Journal of International Finance Markets Institutions and Money 9(1), 61-74.

Hamburger, M.J. & Kochin, L.A. (1972). Money and stock prices: The channels of influences, Journal of Finance, 27 (2), 231-249.

Harris, R. & Sollis, R. (2003). Applied Time Series Modelling and Forecasting, Wiley: Chichester, UK.

Homa, K.E. & Jaffee, D.M. (1972). The supply of money and common stock prices, Journal of Finance, 26 (5), 1045-1066.

Johansen, S. (1991). Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models, Econometrica, 59 (6), 1551-1580.

Kwon, C.S. & Shin, T.S. (1999). Cointegration and causality between macroeconomic variables and stock market returns, Global Finance Journal, 10 (1), 71-81.

Lai, S., Cheng, T.Y., Li, H.C. & Chien S.P. (2013). Dynamic Interactions among macroeconomic variables and stock indexes in Taiwan, Hong Kong, and China", Emerging Markets Finance & Trade, 49, Sep2013 Supplement, 213-225.

Lee, C.H., Doong, S.C. & Chou, P.I. (2011). Dynamic correlation between stock prices and exchange rates, Applied Financial Economics, 21(11), 789-800.

Lutkepohl, H., Saikkonen, P. & Trenkler, C. (2001). Maximum eigenvalue versus trace tests for the cointegrating rank of a VAR process, Econometrics Journal, 4 (2), 287-310.

Miyakoshi, T., Li, K.W. & Shimada, J. (2014). Rational expectation bubbles: evidence from Hong Kong's sub-indices, Applied Economics, 46 (20), 2429-2440.

Montes, G.C. & Tiberto, B.P. (2012). Macroeconomic environment, country risk and stock market performance: Evidence for Brazil, Economic Modelling, 29 (5), 1666-1678.

Naifar, N., & Al Dohaiman, M. S. (2013). Nonlinear analysis among crude oil prices, stock markets' return and macroeconomic variables. International Review of Economics & Finance, 27, 416-431.

Nie, T.H. (1997). Stock market integration in Hong Kong and China, Journal of Contemporary China, 6 (16), 487-512.

Patra, T. & Poshakwale, S. (2006). Economic variables and stock market returns: evidence from the Athens stock exchange, Applied Financial Economics, 16 (13), 993-1005.

Patra, T. & Poshakwale, S. (2008). Long-run and short-run relationship between the main stock indexes: evidence from the Athens stock exchange, Applied Financial Economics, 18 (17), 1401-1410.

Pesaran, H.H. & Shin, Y. (1998). Generalized impulse response analysis in linear multivariate models, Economics Letter, 58, 17-29.

Reimers, H.E. (1992). Comparisons of tests for multivariate cointegration, Statistical Papers, 33(1), 335-359.

Su, Q., Chong, T.L & Yan, K.M. (2007). On the convergence of the Chinese and Hong Kong stock markets: a cointegration analysis of the A and H shares, Applied Financial Economics, 17 (16), 1349-1357.

# APPENDIX 1

## Influence of HSCEI and HSCCI on the HSI

This part measures the proportion of total market capitalization held by the constituents of the Hang Seng China Enterprises Index (HSCEI) and the Hang Seng China-Affiliated Corporations Index (HSCCI), in the Hang Seng Sub-indexes.

**Table A1. Influence of HSCEI and HSCCI on the HSI in terms of Market Capitalization**

| **Panel A: Commerce and Industry Sub-index (C&I)** | | | | |
|---|---|---|---|---|
| | **Market Cap. (HK$B)** | | **Market Cap. (HK$B)** | |
| [0013] Hutchison | 382.42 | [0494] Li & Fung | 65.96 | |
| [0019] Swire Pacific 'A' | 92.01 | [0700] Tecent | 1,065.17 | |
| [0027] Galaxy Ent | 203.97 | [0762] China Unicom | (R) 254.06 | |
| [0066] MTR Corporation | 184.06 | [0857] PetroChina | (H) 170.69 | |
| [0135] Kunlun Energy | (R) 57.48 | [0883] CNNOC | 449.15 | |
| [0144] China Mer Hldings | (R) 68.42 | [0941] China Mobile | (R) 1,848.82 | |
| [0151] Want Want China | 132.22 | [0992] Lenovo Group | (R) 117.75 | |
| [0267] CITIC | (R) 334.20 | [1044] Hengan Int'l | 96.97 | |
| [0291] China Resources | (R) 37.77 | [1088] China Shenhua | (H) 76.64 | |
| [0293] Cathay Pac Air | 68.29 | [1880] Belle Int'l | 69.33 | |
| [0322] Tingyi | 95.60 | [1928] Sands China | 330.34 | |
| [0386] Sinopec Corp | (H) 153.85 | [2319] Mengniu Dairy | 59.24 | |
| Total market capitalization in C&I (HK$B) | | | 6,414.41 | |
| Market capitalization of HSCEI constituents in C&I (HK$B) | | | 401.18 | 6.254% |
| Market capitalization of HSCCI constituents in C&I (HK$B) | | | 2,718.50 | 42.38% |
| **Panel B: Finance Sub-index (FIN)** | | | | |
| | Market Cap. (HK$B) | | Market Cap. (HK$B) | |
| [0005] HSBC | 1,442.37 | [1398] ICBC | (H) 459.14 | |
| [0011] Hang Seng Bank | 243.19 | [2318] Ping An | (H) 275.56 | |
| [0023] Bank of E Asia | 71.23 | [2388] BOC Hong Kong | 274.89 | |
| [0388] HKEx | 204.53 | [2628] China Life | (H) 199.05 | |
| [0939] CCB | (H) 1,442.50 | [3328] Bank COMM | (H) 233.88 | |
| [1299] AIA | 520.35 | [3988] Bank of China | (H) 343.69 | |
| Total market capitalization in FIN (HK$B) | | | 5,710.38 | |
| Market capitalization of HSCEI constituents in FIN (HK$B) | | | 2,953.82 | 51.73% |
| Market capitalization of HSCCI constituents in FIN (HK$B) | | | 0 | 0% |

| | | | | |
|---|---|---|---|---|
| **Panel C: Properties Sub-index (PROP)** | | | | |
| | Market Cap. (HK$B) | | Market Cap. (HK$B) | |
| [0001] Cheung Kong | 305.04 | [0083] Sino Land | 75.68 | |
| [0004] Wharf (Hldgs) | 167.87 | [0101] Hang Lung Prop | 97.33 | |
| [0012] Henderson Land | 155.12 | [0688] China Overseas | (R) 188.00 | |
| [0016] SHK Prp | 320.13 | [0823] Link REIT | 111.68 | |
| [0017] New World Dev | 77.12 | [1109] China Res Land | (R) 116.04 | |
| Total market capitalization in PROP (HK$B) | | | 1,614.01 | |
| Market capitalization of HSCEI constituents in PROP (HK$B) | | | 0 | 0% |
| Market capitalization of HSCCI constituents in PROP (HK$B) | | | 304.04 | 18.84% |
| **Panel D: Utilities Sub-index (UTI)** | | | | |
| [0002] CLP Holdings | 165.61 | [0006] Power Assets | 161.03 | |
| [0003] HK & China Gas | 185.01 | [0836] China Res Power | (H) 97.62 | |
| Total market capitalization in UTI (HK$B) | | | 609.27 | |
| Market capitalization of HSCEI constituents in UTI (HK$B) | | | 97.62 | 16.02% |
| Market capitalization of HSCCI constituents in UTI (HK$B) | | | 0 | 0% |
| Total market capitalization in HSI (HK$B) | | | 14,348.07 | |
| Market capitalization of HSCEI constituents in HSI (HK$B) | | | 3,452.62 | 24.06% |
| Market capitalization of HSCCI constituents in HSI (HK$B) | | | 3,022.54 | 21.07% |

*Notes: The calculation was based on the market capitalization as of 15 December 2014, published in the aastocks.com. The stock code on the Hong Kong Exchange is in []. (H) and (R) denote the current constitutes of Hang Seng China Enterprises Index and Hang Seng China Affiliated Corporations Index, respectively.*

**Biography of author(s)**

**Hok-Fu Wu**
Crowe (HK) Global Corporate Advisory Limited, Hong Kong.

He obtained a Bachelor of Arts in Economics and Finance (First-class Honours) from Hong Kong Shue Yan University and a Master of Science in Applied Economics (Distinction) from the City University of Hong Kong. He is working as a manager in the valuation team of Crowe (Hong Kong) Global Corporate Advisory Limited.

**Kai-Yin Woo (Associate Professor)**
Department of Economics and Finance, Hong Kong Shue Yan University, Hong Kong.

He is an Associate Professor at the Department of Economics and Finance of Hong Kong Shue Yan University. He obtained PhD at the University of Stirling, Scotland. His research interests include the economics of finance and applied econometrics. His research topics include bubble testing, investigation of price convergence and purchasing power parity, and empirical study on the Belt and Road Initiative and the Greater Bay Area of China. He has published journal articles in Economics Letters, Applied Economics, Economic Modelling, Journal of Macroeconomics, Singapore Economic Review, International Review of Economics and Finance, Advances in Decision Sciences, Applied Economics Letters, Empirical Economics and so on.



**Peer-Review History:**
This chapter was reviewed by following the Advanced Open Peer Review policy. This chapter was thoroughly checked to prevent plagiarism. As per editorial policy, a minimum of two peer-reviewers reviewed the manuscript. After review and revision of the manuscript, the Book Editor approved the manuscript for final publication. Peer review comments, comments of the editor(s), etc. are available here: https://peerreviewarchive.com/review-history/12411F-CH5

# INDEX

**P**



**R**



**S**



**T**



**V**



---

**London Kolkata Tarakeswar**

**India:** Guest House Road, Street no - 1/6, Hooghly, West Bengal, India (Reg. Address),
Diamond Heritage Building, 16, Strand Road, Kolkata, 700001 West Bengal, India (Corporate Address),
Tele: +91 7439016438 | +91 9748770553, Email: director@bookpi.org,
**(Headquarters)**

**UK:** 27 Old Gloucester Street London WC1N 3AX, UK,
Fax: +44 20-3031-1429, Email: director@bookpi.org,
**(Branch office)**